INTERMODULAR COMMUNICATION WITH ADDRESSING IN PLANAR ARRAYS OF MODULES CONNECTED WITH ONE WAY-CHANNELS
(Selcuk Networks)

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INTERMODULAR COMMUNICATION WITHOUT ADDRESSING IN PLANAR ARRAYS OF MODULES CONNECTED WITH ONE-WAY CHANNELS

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ABSTRACT

This research aims at determining the structure and operating rules of networks that allow intermodular communication without resort to addressing.* (Addressing is defined here as the capacity to connect with a non-adjacent module irrespective of its content.) It is desired that an information request or a general message emanating from any module propagate to the network reasonably rapidly; and that upon receiving a request any module, if it so wants, be able to start a response which will travel a unique and reasonably short path to the module that made the request.

The study covers square and hexagonal arrays. It is shown that when the channels connecting modules are two-way, intercommunication without addressing is easily achieved when modules on the path of a response apply simple rules locally and in total ignorance of what other modules are doing. Propagation rates are proportional to the square of time and response paths are shortest paths.

A priori it might be thought that if the channels were made one-way, propagation would be slowed down and response routing would be very cumbersome, if it were possible at all. Comparative graphical analyses in square and hexagonal networks reveal that for certain arrangements of one-way channels, propagation rates and response paths are obtained which compare very favorably with corresponding arrays with two-way channels. Furthermore in such one-way channel networks intermodular communication is achieved with significantly less logic and memory in each module. For large networks these savings become considerable.

Graphical analyses are again utilized to identify determinants of efficiency in networks with one-way channels. It is shown that if channels having the same orientation i.e. either towards the module or away from the module, are made adjacent efficiency is enhanced. Arranging channels such that flows in straight lines become possible also improves efficiency.

*Patent is pending for the communication method described in this paper.

The conclusions are relevant to communication problems where it is desired to allow intermodular communication (e.g. connecting a large number of computers). They are also relevant to computation and in particular to parallel processing and to associative memories. When designing parallel machines like ILLIAC, and content-addressable computers, the schemes suggested in this paper dispensing with central control units, and the possibility of logic and memory saving through use of one-way channels as demonstrated here, should be carefully considered.

The results of this research may be found to be relevant to understanding nervous systems of biological beings. The neuron, the fundamental unit of nervous systems, appears to be much more complex than a switch. The nervous systems are constructed with one-way channels; the dendrites normally bring in messages, the axons carry them away. Interestingly "adjacence of channels having the same orientation" abounds in that normally dendrites are adjacent. The branches of the axon are obviously adjacent. If communication in the nervous system were indeed taking place along lines suggested here, then this would explain the observed absence of a central control unit for memory, and the apparent insensitivity of memory to destruction of large portions of neural matter as well as memory traces being dispersed throughout the network.

Implications of communication without addressing merit extensive investigation. Some suggestions for further research are made in this study.

CHAPTER I

INTRODUCTION

Motivation

The research reported here stemmed from the author's research on the implications of non-directed associative thought processes for intelligence.[1*] Various hypotheses proposed in this regard rested on the question of possible bases of human memory. My previous experience with associative and distributed-logic, distributed-memory computer schemes motivated me to postulate that a memory unit in the brain could be viewed as an intercommunicating group of neurons.[2] Each memory task would presumably have an associated assembly of neurons.

During many years of intensive research Lashley was unable to find memory localization in the brain or a central mechanism responsible for memory.[3] Removal of sizable portions of the cortex did not seem to impair memory. Teuber encountered similar results.[4,5] During this decade, however, very ingenious experiments on RNA production in neural tissue as well as enzyme systems have suggested that memory might in part be localized in macromolecules (Schmitt,[6] Kasha,[7] Augenstine,[8] Rich,[9] Morrell[10]). Postulating that the basis of memory is an intercommunicating assembly of memory modules (neurons?) would appear to solve the apparent paradox between Lashley's findings and the findings of macromolecular biologist.[11]

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[*]Superscripts refer to notes at the end of this chapter.

The fundamental problem with this postulate is how can a few neurons from among the billions intercommunicate? Two basic solutions suggest themselves: using addressing schemes and using schemes which do not require addressing. By addressing scheme I mean the ability to locate any module (neuron) irrespective of content. Addressing schemes would be tremendously complicated for networks having billions of modules. This reasoning led me to believe that in unidirectional networks* (of which neural networks are examples) the intercommunication problem could be solved without addressing schemes. The study reported here resulted from that conviction. However, I have divorced the problem from its neurophysiological context and treated it in its own right.

Formal Statement of the Problem

Given a large group of modules how can they be interconnected so that anyone of them can initiate an information request (general message) which will spread to the rest of the modules and so that when any module wants to respond to the general message its response will travel a unique and a reasonably short path to the module that initiated the request? There are to be no cycles in the response path. Each module is to be totally ignorant about the locations of the others or what they are doing or will do. Therefore the module initiating the response can in no way control the path so as to assure that the response gets to the general message source. That is to say intermodular communication is to achieved without addressing i.e. without the capability of locating a module irrespective of its content.

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*By unidirectional I mean networks with one-way channels (Selcuk networks).

The Approach in this Study

This paper will propose two types of networks where no addressing provisions are made and will investigate the rules if any for inter-modular communication. In both types the modules will be arranged in arrays i.e. each module will be connected to some or all of its immediate neighbors.

In type 1, channels that connect any two modules will operate in two directions. Such networks will therefore be called bi-directional networks. In the uni-directional networks, channels will be allowed to transmit in one direction only, either towards the module or away from it. Therefore, while module A will be able to send messages to its neighbor B through channel AB, it will not be able to receive messages from B on the same channel. B's messages intended for A must go through other modules.

The severe constraints on flow directions would almost seem to make the uni-directional network inferior to the bi-directional one.[14] A comparative study undertaken in this research will explore the a priori hypothesis that making channels one-way would make a network somehow inferior to those having two-way connections.

The bi-directional and the uni-directional networks will be comparatively investigated through graphical analysis and cut-and-try methods to find operating rules.[15] In order to simplify the problem as much as possible so as to generate fundamental results I will concentrate on networks in which each module has the same number of channels. Channels will be equi-length.

In two dimensions, only the triangle, the square and the hexagon satisfy this property. The proof can be found in Ore.[16] In three dimensions there are only five configurations: tetrahedron, cube, dodecahedron, octahedron and icosohedron.[17] The analysis in this paper is confined to two dimensional graphs only, because they represented a very rich area of research and because study of three dimensional figures appeared to be enormously complicated.

Consider the triangle. It has the following possible arrangements.

Module (A) and (B) cannot intercommunicate. So they are out. Module (C) has just one input channel. Patterns in the arrival channel of the messages do not allow it to make any output channel decisions. In module (D) no matter how a message arrives it is transmitted on the same channel. For routing to be possible module (C) and (D) must be considered together, i.e. as

This is equivalent to a square configuration. Therefore, triangular networks can be ruled out. Hence only the square and the hexagonal modules remain to be studied.

Review of Previous Research

The problem of intermodular communication appears to be common to computer science, communication theory, automata theory, human communication networks, and neurophysiology.

In the computer area parallel processors, content-addressable associative memories, distributed-logic distributed memory machines, and computer networks involve communication between units. It appears that providing for such communication without addressing has not been explicitly treated in anyone of these fields. Consider the ILLIAC-IV machine which is a parallel processor.[18,19] Identical processing elements each having upwards of 4000 bits of core storage and arithmetic capabilities are arranged in a square array. Modules i.e. processing elements can communicate with their immediate neighbors only. Intercommunication between non-adjacent units is mediated by the central unit which determines the path and gives routing instructions to modules lying along the path, which is unlike our proposal.[20]

In associative memories the purpose is to provide for retrieval on the basis of content, i.e. without addressing. A good description is provided by McKeever.[20] "Content addressability" in a sense is part of the problem treated in this paper. However, the proposal to attain it as embodied in associative memories is quite different from ours. The memory is an array of binary cells. Each row stores a word. The search word is placed in the "register" which is the top most row. Each bit is

then relayed down the corresponding column. If the content of a cell which lies along the column line of a bit does not match that bit, a signal is propelled along the horizontal line. Signals are summed along the rows. Hence if the adder corresponding to a row has zero that row's contents match the search word. The scheme does not really provide intermodular communication in general. At best it can be said that one row (the register row) is able to communicate with the other rows. The reader desiring to learn about the variations and refinements of the basic associative memory can refer to McAteer, et al,[21] to Seeber,[22] Seeber and Lindquist,[23] and to Ewing and Davis.[24]

In distributed-logic distributed-memory computers as described by Lee[25] while intermodular communication problem definitely exists it does not appear that a solution to it without addressing other than the ILLIAC-IV scheme has been considered. One emerges with the same conclusion after examining the works on cellular arrays. See for instance Minnich[26] and Canaday.[27]

Apparently the central problem of this study is original. Verbal communications with Drs. Licklider and Jones of Project MAC of MIT, Prof. Ness of MIT, and Drs. Bobrow and Alkind of Bolt, Beranek and Newman Inc. supported this conclusion.

Network communication among human subjects was first researched by Bavelas[28] then by Guetzkow and Simon[29] and more recently by Cohen.[30] It appears that these studies considered networks with five nodules only and then only two-way channels.

In the automata theory field modular arrays as pattern recognizers have been studied by Unger[31] and more recently by Beyer.[32] Beyer considers the problem of recognition of topological invariants by modular arrays. A "finite rectangular two-dimensional iterative array of deterministic finite state automata" is proposed as pictured in figure 1.1.

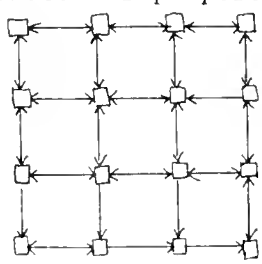


Figure 1.1 Modular Array Proposed by Beyer

All of the modules in the array are assumed to be identical and each is connected to its four neighbors. The array functions synchronously. Beyer does not recognize or treat the case of uni-directional modular arrays. Communication between any two modules without addressing is not provided. No algorithm is stated which would accomplish this in a way other than scanning all the rows or all the columns successively.

It appears that the case of one-way connections between modules has not been proposed or researched by Unger either.[31] Professors Minsky, Pappert and Hennie of Project MAC at MIT indicated that the topic of this paper has not been studied by automata theoreticians, at least not in the comparative manner followed here.[33]

The most natural context of our problem is neurophysiology. The neuron, presumed to be the basic unit, has one-way channels. Dendrites are those leads that normally receive excitation and conduct it to the cell body; the axons are those leads that normally transmit excitation

away from the cell body. The arrangement of the neuron depends on where it is located in the nervous system.

As Belmont Farley points out, "it has been widely assumed, particularly in engineering circles, that neuron action is all-or-none, and that its function is represented satisfactorily by a simple logical element."[34] When neuron is assumed to be such a simple element, the emphasis shifts to the behavior of the network. Interneuronal communication remains an irrelevant question. However, "it is clear that the neuron is much more complicated than a single gate."[35] Recent studies on macromolecules as possible bases of memory have yielded positive results. (For example, see Schmitt.[36]) If this is so then each neuron may be harboring a large memory capacity. In the light of such results interneuronal communication is a very relevant and important topic to pursue.

The extent of exploration done in this paper is as yet indirectly pertinent to the topic of neuronal intercommunication. Many properties and parameters of the neuron do not appear in my regular arrays. My chief interest in considering neurophysiology here is to point out that it is an appropriate context for the problem at hand and determine the extent of research done on communication between any two sites in a network without resorting to addressing. For this I relied chiefly on Rosenblatt's well balanced review of neural networks through 1961 in his Principles of neurodynamics.[37] I was unable to find any allusion to my problem. Rosenblatt's perceptron theories described in detail in the same book do not include a treatment of intermodular communication without addressing in

large nets. My limited knowledge of neurophysiology and the extensiveness of recent work in this area did not permit me to peruse the literature on neural networks since 1961. Instead I relied on word of authority. Professor Murray Eden at MIT indicated that the problem of intermodular communication in the manner proposed in this paper was not treated in neural network studies.[38]

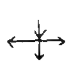
For the communication theory area Fano's authoritative book on the subject, The Transmission of Information, was consulted.[39] No explicit mention or treatment of the problem of this study was encountered. However, my limited sophistication in this area caused me to seek word of authority. Professor Shannon of MIT indicated that the findings reported here are interesting and original in the communication theory area.[40]

Taxonomy

When channels are one-way, for a given geometric configuration several arrangements of channels are possible. All possible arrangements with their arbitrarily assigned class names are shown below.

Square:

Class A  Class D 

Class B  Class E 

Class C  Class F 

Hexagon:

Class A 
Class B 
Class C 
Class D 
Class E 
Class F 
Class G 
Class H 
Class I 
Class I' 
Class J 
Class K 
Class L 
Class M 

For a network to consist of elements of one class, the number of input channels and the number of output channels in the module must be equal. Only square class C and D, and hexagon class D, I, I', and K modules satisfy this requirement. I have concentrated on these classes because I wanted to study single class networks to see if arrangements of one-way channels influenced the operational characteristics of networks. Later, however, mixed graphs had to be considered.

Possible Contexts for Intermodular Communication Without Addressing

Both communications and computation areas provide contexts for the problem at hand. And both contexts find their own context in information systems. In the communications area, whenever it is desired to connect a large number of units (people, computers, telephones) so as to allow conversations among units, achieving this without addressing needs to be

examined. For instance an organization may have upwards of a thousand computers spread across the globe, each computer specializing in the problems of its immediate area. Problems being interdependent it may be desired to give each computer the capability to interrogate all the other computers as frequently as needed on particular problems and topics and then to communicate with the ones having pertinent information. As another example, consider a symposium attended by many hundreds of professionals. Normally each participant would have one or more topics of interest to him that he would like to discuss with those concerned with those topics. It rarely is possible to sort out the people having common interests or complementary information. Instead a very few participants expose their views and again very few reveal their interests or thoughts on the presentation through questions and direct contact. If each participant could be provided with a simple input-output device and if these devices could be connected so as to allow interunit communication the participants could fairly rapidly air out their questions and thoughts. If need be the group could then break up into discussion groups. A further example would be large organizations where company-wide consultation of conferencing might be either an important part of operations or helpful for improving the efficiency of the operations. The executives could be connected to devices in a network which provided for intercommunication. In all these instances possibility of providing for intercommunication without addressing and also doing this with one-way channels should be studied.

In the computation area there have been efforts to design parallel machines like the ILLIAC. As mentioned previously possibility of providing communication among modules in such machines without addressing and also doing the same with one-way channels have not been considered. If large machines of this kind are comtemplated the results of this research would be relevant.

Associative memory computers allow content addressability. In the currently proposed shcemes search is initiated from one location. If it is desired to allow for initiation of content-addressed search from any site in memory, constructing modular arrays perhaps with one-way channels that achieve intercommunication without addressing needs to be studied.

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CHAPTER II

INTERCOMMUNICATION IN SQUARE ARRAYS

Overview

Modular arrays arranged in a square fashion and having two-way channels are studied. The analysis is repeated for arrays with one-way channels. Propagation and response times are compared. A priori it is hypothesized that networks with one-way channels will be slower than bidirectional networks in propagating the information requests. It is further hypothesized that response routing is more complex when channels are one-way. The hypotheses are shown to be untrue for certain types of uni-directional networks.

Introduction

In a square arrangement each module is connected to four neighbors as shown in Figure 2.1. When the channels are two-way, messages can be received from and transmitted to all four neighbors. Upon receiving a request (here called a general message) if a module propagates it to its neighbors who in turn pass it on to their neighbors and so on, the request will reach all the modules in the network. The rate of this propagation must and can be computed to assess feasibility for a particular communication problem.

When the channels are one-way only some of the neighbors can send messages to a particular unit and only some of the neighbors receive messages from that unit. The freedom of transmission is reduced. In the case of networks constructed with class C or class D modules, freedom is reduced by exactly fifty percent. A priori it might be thought that because of the restrictions on flow directions unidirectional networks are slower in propagating a general message than the bidirectional ones.

Although propagation rates are important, the essence of our inquiry is to find rules which when applied by each module locally and independently will route a response from any module to the general message source without cycles and via a reasonably short path.

Whatever the rules might be, a priori those for the unidirectional network may be thought to be more complex. Consider the simplest case where a response is to be routed to an immediate neighbor. When channels operate both ways this is no problem at all. When a channel can conduct one way, to route a response to a neighbor module that has just sent a request, a module must go through several other ones. Since it has no control over the intermediary modules it is not clear how a response can get even to a neighbor. When channels can operate both ways this is no problem at all. When a channel can conduct one way, to route a response to a neighbor module that has just sent a request, a module must go through several other ones. Since it has no control over the intermediary modules it is not clear how a response can get even to a neighbor.

Operating Rules for the Bi-directional Square Network

A module in a bi-directional network can receive and send a message on the same channel. When a message arrives on a channel, the module connected to the other end has already received the message. If the message is a general message which is to be propagated to all the neighbors, it would be wasteful to propagate along the channels that brought it in the first place. Otherwise, when a unit sends a general message, a short time later it will come back to it. Lest this back and forth volleying continue forever, stopping rules must be introduced someplace. The following rule is proposed:

When a bi-directional module receives a general message, it propagates it on all channels except the one(s) that had brought the message in.

The propagation pattern when this rule is used, is shown in Figure 2.1. The intersections represent the modules. Horizontal and vertical lines are the channels whereas the heavy diagonal lines are iso-temporal propagation lines. Notice how the latter form concentric squares. Arrow heads mark the channels that brought the general message first while the numbers show the time elapsed since the initiation of the request. The general message originates at the center of the diagram. The propagation rate can be inferred by simply counting the number of modules reached at each time unit. The results of such a count are shown in table 2.1, as well as the formulas obtained by induction.

Figure 2.1 Propagation in the Bi-directional Square Network.

Time	Number of Additional Modules Covered (Propagation Rate)	Number of Total Modules Covered (Cumulative Propagation)
1	4	4
2	8	12
3	12	24
4	16	40
5	20	60
6	24	84
.	.	.
.	.	.
.	.	.
n	4n	$2n^2 + 2n$

Table 2.1 Propagation Rates in the Square Bi-directional Network

Upon receiving the general message and passing it along, if a module wants to respond to it and have its response reach the source of the general message as rapidly as possible, it should send it out on the channel through which the general message first arrived. The next module receiving the response should transmit again on the channel upon which the general message originally arrived and so on. This algorithm will carry the response through the shortest path. The time for the response to get to the request source will be the same as the time for the general message to reach the responding module.

Networks having two-way channels solve the basic problem that any module be able to propagate rapidly a message to the network and then receive a response from any module, all this in total ignorance about the contents and locations of other modules. The cumulative propagation is proportional to the square of time, which is quite fast. For instance in a hundred time units (which could be hundred microseconds) 20,200 modules would be covered.

Uni-directional Channel Network:
The Class C Configuration

In the networks having one-way channels, while module A can speak to module B, B cannot speak to A directly but must send its message to C who sends it to D who sends it to A. Note, however, because a module operates in ignorance of what other modules are doing, module B has no control over the path of the response intended for A. For all it knows module C might be routing messages to module E.

For instance, consider the network consisting of class C elements only or appearing in Figure 2.2. Ignore the rectangles. The x-coordinate is indicated above the graph while the y-coordinate is on the left. Suppose (8,7) wants to communicate with (8,6). It can simply send a message up its northern channel. If (8,6) wants to reply to (8,7), its message must go to (7,6) who must route it to (7,7) who in turn must relay it to (8,7).

It should be noticed that when a message is propagated to the entire network from a source the general message arrival direction is not the same even for the modules in the same quadrant. Quadrants are defined with respect to the source. In Figure 2.2 the source is (8,8). While (7,6) receives the general message from the right, (7,5) receives it from above and from the left. (13,8) which is in another quadrant also receives it from above and from the left.

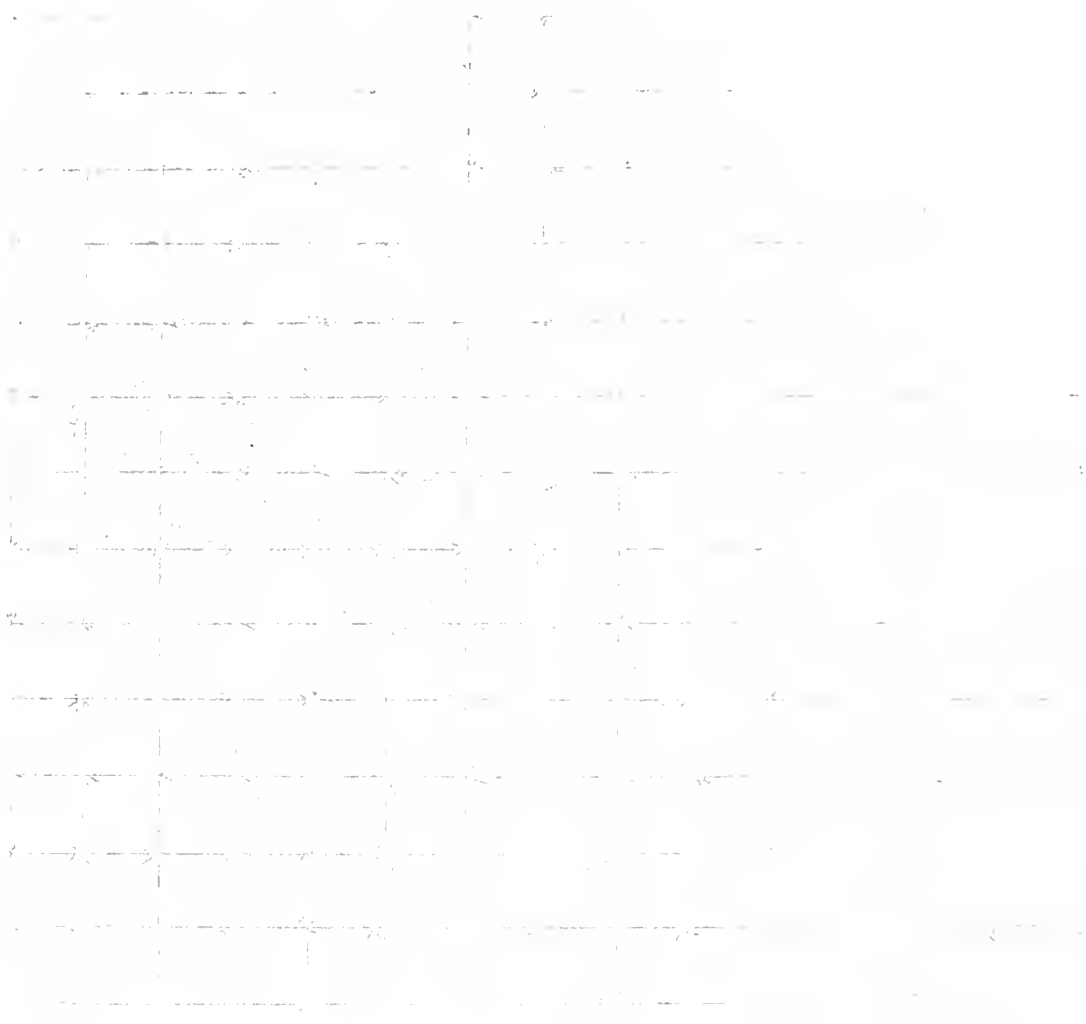


Figure 2.2 Propagation in the Uni-directional Class C Network.

All these properties might lead one to expect networks with one-way channels to be inefficient and cumbersome and incapable of response routing with simple local rules.

Consider the propagation pattern appearing in Figure 2.2. The solid lines indicate isotemporal propagation; that is, all modules on the same rectangle receive messages simultaneously. Although the pattern is complex it is regular. The isotemporal lines from concentric rectangles with arms extending two layers in. The propagation rate can be determined by counting the number of modules associated with each isotemporal line. The results are in table 2.2. The irregularity at time = 4 occurs because (8,8) would normally be covered at time four by an arm reaching inward. Compare with (7,9). However (8,8) is the source and, therefore, was covered at time zero. Then on there is no irregularity.

Time	Number of Additional Modules Covered	Number of Total Modules Covered
1	2	2
2	4	6
3	8	14
4	11	25
5	16	41
6	20	61
7	24	85

Table 2.2 Propagation in Square Uni-directional Class C Network

The propagation rate increases by four modules except at t = 1 and t = 4. Therefore, the propagation rate is

$$\text{P. R.} = \begin{cases} 2 & n = 1 \\ 11 & n = 4 \\ 4(n-1) & n > 1,\ n \neq 4 \end{cases} \qquad n = \text{time}$$

and the total propagation is

$$\text{T. P.} = \begin{cases} 2n^2 - 2n + 2 & 1 \leq n < 4 \\ 2n^2 - 2n + 1 & n \geq 4 \end{cases}$$

To compare the cumulative propagation in the two types of networks the limit of total propagations can be computed, i.e.

$$\lim_{n\to\infty} \frac{\text{T.P.}_{BD}}{\text{T.P.}_{UD}} = \lim_{n\to\infty} \frac{2n^2 + 2n}{2n^2 - 2n + 1} = \lim_{n\to\infty} \frac{n + 1}{n - 1 + \frac{1}{2n}} = 1$$

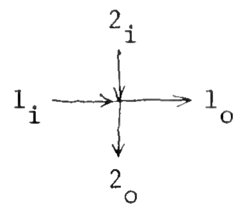
where the subscript BD refers to the bidirectional and UD to the unidirectional.

The limit approaches "1" very rapidly. In 50 time units unidirectional propagation will cover about $\frac{49}{50}$th of modules covered in the bidirectional network. Therefore, for simplicity I have assumed that $2n^2$ represents total propagation in both the uni-directional class C network and the bi-directional network.

Response Routing in the Uni-directional Network

Is it possible for each module in the class C network to route a response only on the basis of how the corresponding general message arrived, so that the response reaches the request source? If networks having unidirectional channels do not have this capability then the only way they can provide intermodular communication is through always broadcasting the messages. This would be very inefficient in large networks. If this were the case, uni-directional networks would not merit any further study. However, cut-and-try investigations have revealed some very simple rules.

Let the channels be labeled clockwise. That is



With each channel we can associate a binary digit. If it is an input channel, "1" would indicate that that channel brought the message first. Thus if we are referring to the general message, (1,0) would mean that the first channel, 1_i, brought the general message first. If the channel is an output one, "1" indicates that it will carry the response. Thus (0,1) means that the response will be transmitted on output channel 2 only i.e. on 2_o. Therefore, the question is whether the response transmission channel decision can be made a function of only the general message first arrival channel configuration.

Let

i = general message first arrival channel configuration.

j = response transmission channel configuration.

Restated the question is:

j = f(i) ?

The answer is yes, the function being as follows:*

$$j = \begin{cases} (0,1) & \text{if } i = (1,0) \\ (1,0) & \text{if } i = (0,1) \text{ or } (1,1) \end{cases}$$

These rules can be stated as matrix rules. For single arrivals we have:

$$j = \begin{bmatrix} 0\,1 \\ 1\,0 \end{bmatrix} i$$

*The rules are not unique. Other rules are possible. The ones stated here result in the sharpest response return paths and do not cause any cycling.

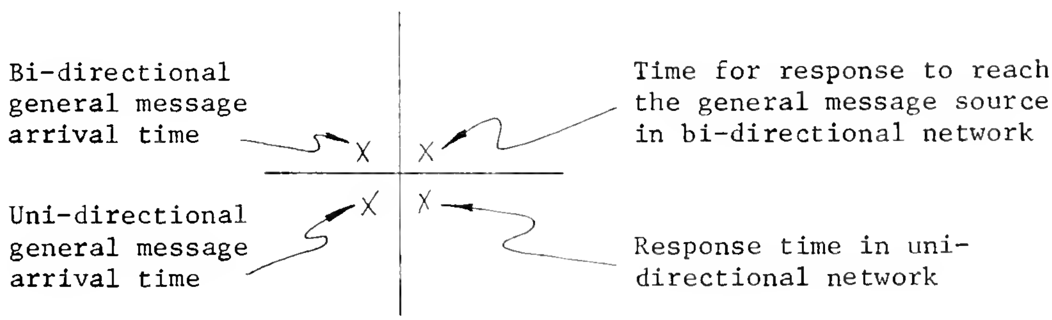
and for multiple arrivals

$$j = \begin{bmatrix} 10 \\ 00 \end{bmatrix} i$$

Application of these rules to the modules in the network resulted in the response pattern shown in Figure 2.3. Modules falling on solid lines transmit responses on channels coinciding with the solid lines. Others join one of the solid lines on either side.

In applications where one has a choice between uni-directional and bi-directional modules one must know at least how uni-directional propagation and response times compare with the bi-directional ones. To this end I computed these times for all modules in both the uni-directional class C network and the bi-directional one.

The results for each module are displayed in Figure 2.4 using the following notation:



Actually the differences between the bi-directional and unidirectional statistics are the meaningful numbers. I have called these "delays" and they are displayed in Figure 2.5 for each module as follows:

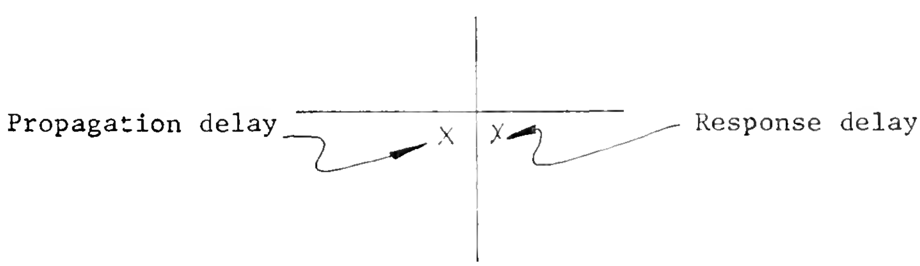


Figure 2.3 Response Return Paths in the Unidirectional Square Class C Network.

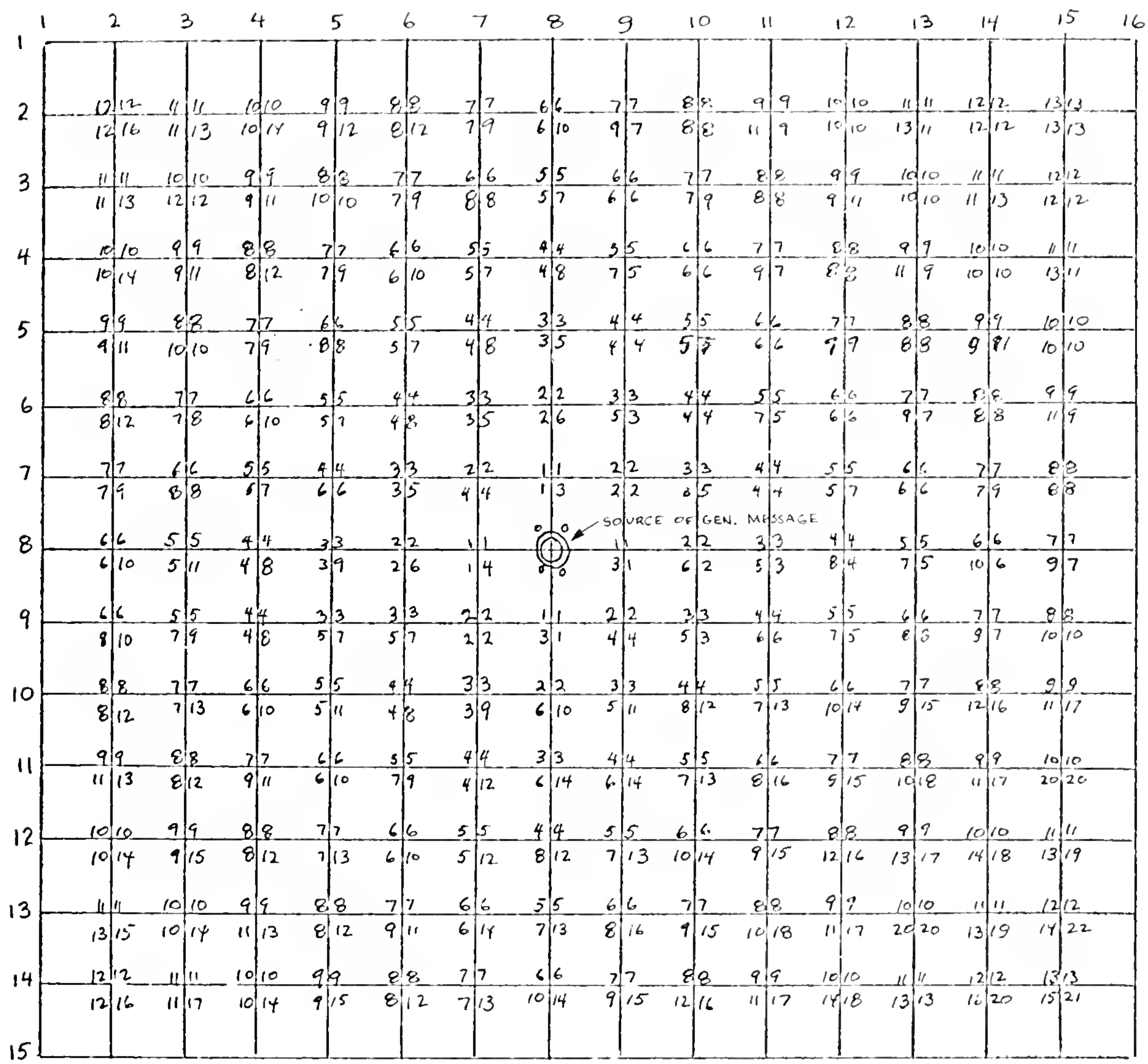


Figure 2.4 General Message Propagation and Response Return Times in Bi-directional Square and in Uni-directional Square Class C Networks.

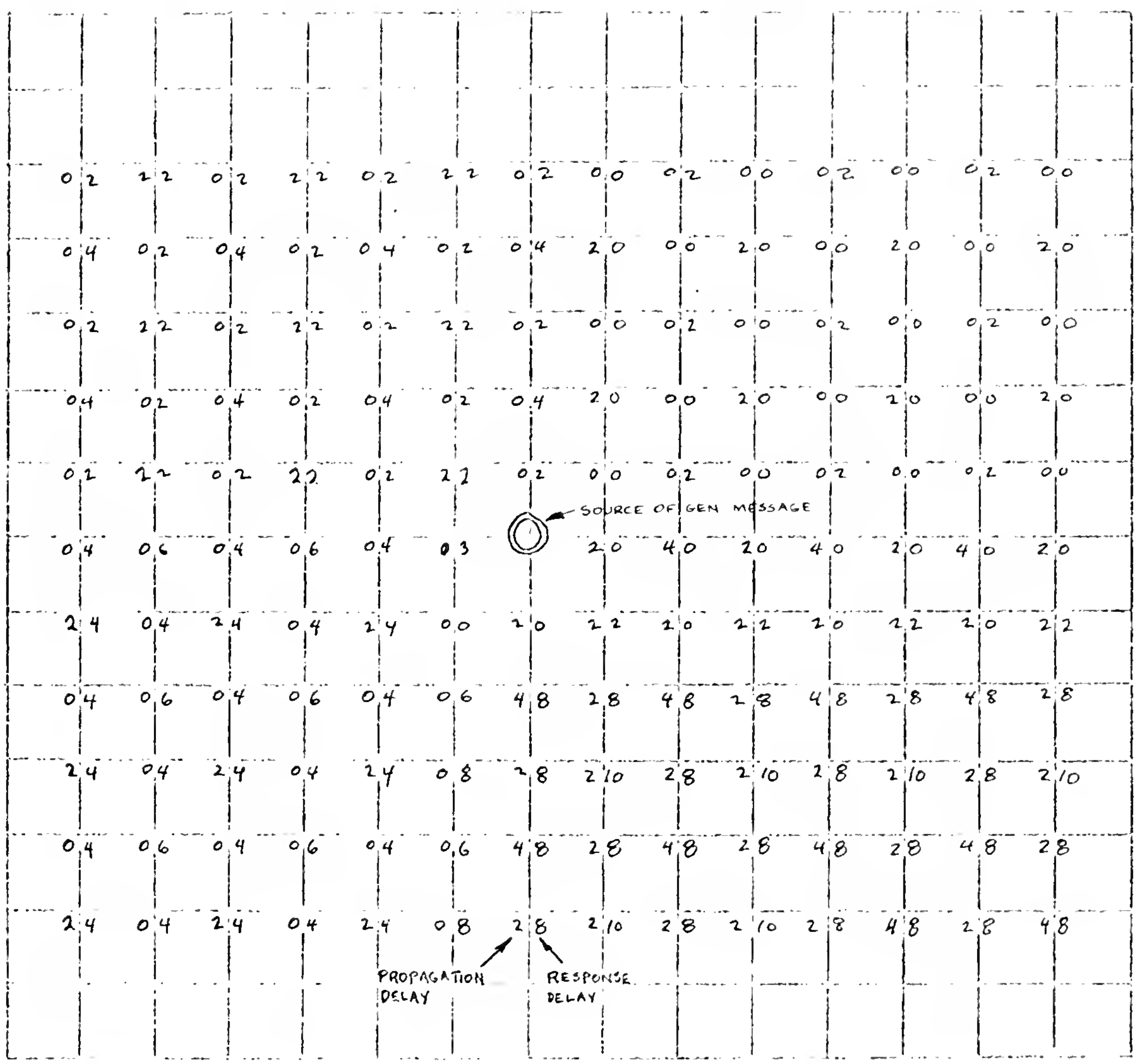


Figure 2.5 Propagation and Response Delays in Uni-directional Square Class C Network.

A visual inspection reveals that the delays remain stable. That is delay ranges and the average delays do not change with the size of the graph. Or more precisely:

$$\sum_{j=1}^{n_k} \Delta_{kj}/n_k = c_k \leq \begin{cases} 2.5 \text{ for propagation} \\ 8.5 \text{ for response} \end{cases}$$

$$\frac{\sum_{k=1}^{4} \sum_{j=1}^{n_k} \Delta_{kj}}{\sum_{k} n_k} = c = \begin{cases} 1.0 \text{ for propagation} \\ 4.0 \text{ for response} \end{cases}$$

where:

Δ_{kj} = delay at module j quadrant k

n_k = number of modules in quadrant k

c_k, c =constants

n_k = 16, 32, 48, ...

k = 1, 2, 3, 4.

The delay statistics for various quadrants are shown in Figure 2.6.

Computational and Memory Requirements; Construction Costs

The a priori hypothesis about the inferiority of uni-directional networks is apparently untrue for uni-directional class C network since this network is almost as rapid in transmitting the general message and the responses as the bi-directional one. However, this alone does not

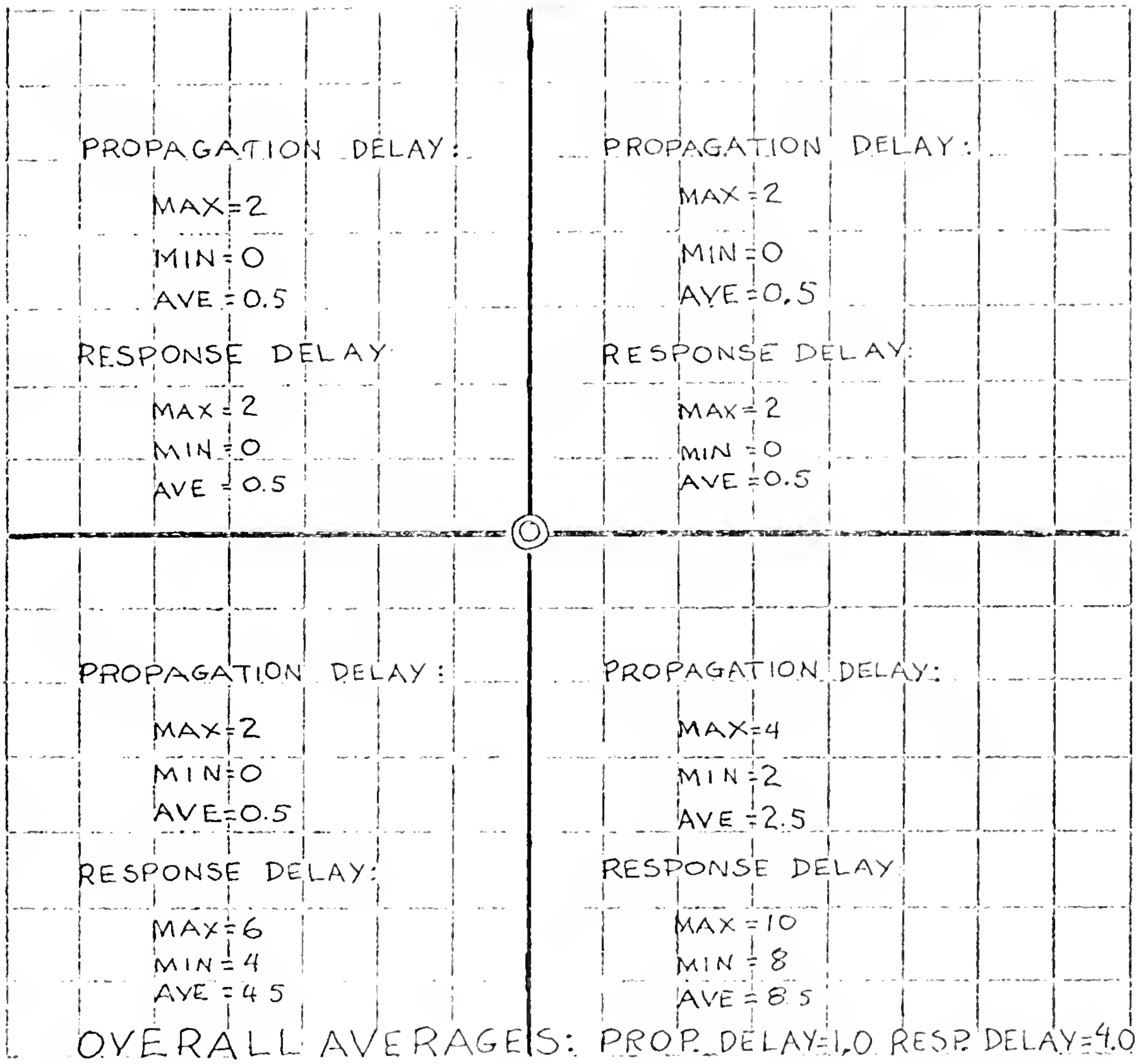


Figure 2.6 Propagation and Response Delays in the Four Quadrants of Uni-directional Square Class C Network.

make the uni-directional superior or preferable. The memory and logic requirements, the number of computations to be performed, and the construction costs, must be comparatively studied.

While it is hard to determine the magnitude the bi-directional channel is more expensive to construct than the uni-directional channel. The bi-directional channel either has to be two uni-directional channels in opposing directions, or a duplex channel, or a channel with temporary storage facilities at both ends. On construction cost considerations, the uni-directional channel network appears superior.

In order to route a possible response, the identities of the channels upon which the general message first arrived must be recorded in both types of networks. In the bi-directional, there are four channels that can bring in a message. Therefore a three-bit word is required. (Two bits are for identifying the channel and one bit is to indicate the absence or existence of a message.) The uni-directional one requires a two-bit word for recording the general message first arrival channel configuration. Hence there is a saving of one bit. Actually the saving may be greater than this.

It is quite possible for the same general message to appear on two channels simultaneously. It is also possible for two different general messages to appear simultaneously. For proper routing of the response the two cases must be differentiated. The bi-directional module must perform three comparison tests for one channel, up to two tests for the second and a maximum one for the third. There could be up to six comparison tests. The minimum would be three, the average around five. All these can easily be ascertained by examining various arrival possibilities in the bi-directional

module. However, the uni-directional requires only one comparison test because messages can appear on two channels only. Comparing the contents of one channel with that of the other would be all that is needed to determine whether the same message had arrived on two channels simultaneously or if the channels brought in two different general messages. Therefore, the uni-directional module is superior as far as processing the incoming general messages are concerned.

In the uni-directional module an incoming general message is sent out on both of the outgoing channels. Thus, no processing is required. In the bi-directional the general message is not to be sent along the channels that brought it in. This stopping rule will require some processing in deciding how to propagate the general message. However there is a stopping rule in the uni-directional network, too. Of the two channels, often one will bring the general message first and the other one will bring it exactly four time units later. This delayed arrival must not be transmitted. The uni-directional stopping rule is simple because the arrival time and arrival channel of the delayed message is exactly predictable. Even if on this count the uni-directional does not hold a clear edge, on all other counts it has an advantage. Also the delayed arrival of the replica of a general message may prove to be very useful for detection of a malfunction of a unit. In the uni-directional network most modules will receive delayed messages and the remaining will receive the same message simultaneously on two channels. This will assure them that the neighbors are functioning.

In the bi-directional network, modules lying along principal axes i.e. (i,10) and (8,j) will receive the general message on one channel while the others will receive it on two. Furthermore as the source changes the principal axes will, too. Hence a unit receiving the general message on one channel only has no way of knowing whether this is because it is on the principal axis or whether it has a malfunctioning neighbor.

Let us compare the response transmission channel rules. A uni-directional module needs three rules only, namely:

$j = (0,1)$ if $i = (1,0)$

$j = (1,0)$ if $i = (0,1)$

$j = (1,0)$ if $i = (1,1)$

The bi-directional module requires eight rules namely:

$j = (1,0,0,0)$ if $i = (1,0,0,0)$	$j = (0,0,1,0)$ if $i = (0,0,1,0)$
$j = (1,0,0,0)$ if $i = (1,1,0,0)$	$j = (0,0,1,0)$ if $i = (0,0,1,1)$
$j = (0,1,0,0)$ if $i = (0,1,0,0)$	$j = (0,0,0,1)$ if $i = (0,0,0,1)$
$j = (0,1,0,0)$ if $i = (0,1,1,0)$	$j = (0,0,0,1)$ if $i = (1,0,0,1)$

Compared to the uni-directional five more rules are needed in the bi-directional. Also each bi-directional rule involves two four-bit words and therefore requires four more bits for storage. Hence assuming at least eight bits per rule the difference in memory space requirements comes to at least $5 \times 8 + 3 \times 4 = 52$ bits per module. When the response does arrive the proper rule must be determined. In the bi-directional module this will involve a search among eight rules each with at least eight bits. In the uni-directional, only three rules each with four bits need to be searched.

Although the figures about bit requirements are at best approximate, the fact remains that uni-directional module requires less memory, logic and computations. Since more computations mean more time, the bi-directional module is actually slower than the uni-directional. These make networks having one-way channels superior to those having two-way channels at least when the task is intermodular communication.

The Uni-directional Class D Network

The class D module is really a slight rearrangement of class C module and it is "nice" bacause it is symmetric. One might, therefore, be intuitively inclined to assume that what holds for class C, will hold for class D and even more strongly because of the nicety of D. This section will investigate class D in some detail.

The propagation pattern for a network containing class D modules only appears in Figure 2.7. The solid lines indicate isotemporal propagation. Notice how a square is surrounded by a rectangle which is surrounded by a square which is surrounded by a rectangle and so on.

The propagation rate can again be determined by counting the nodes associated with isotemporal lines. The results appear in table 2. .

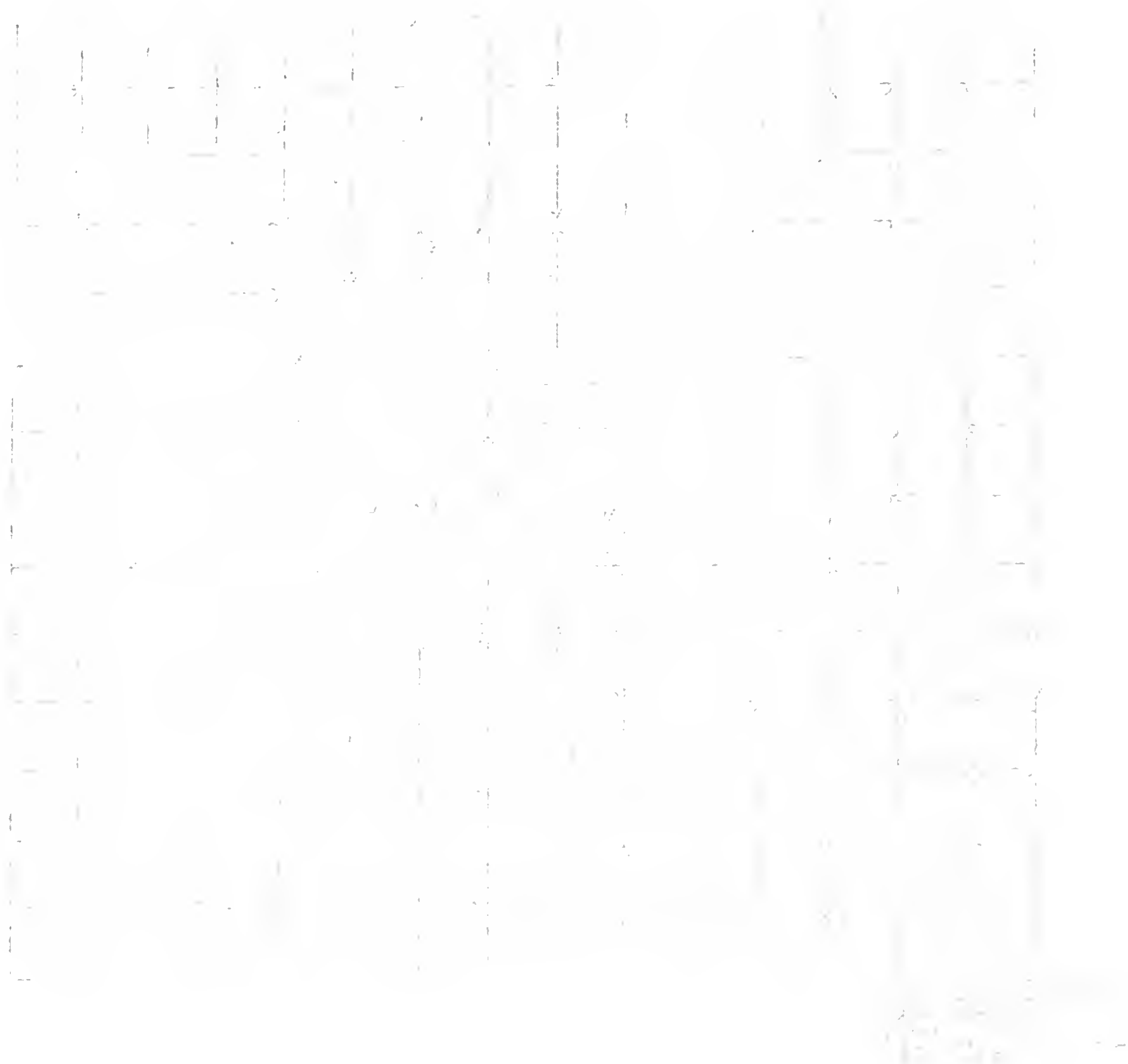


Figure 2.7 Propagation of a General Message in Uni-directional Square Class D Network.

Time	Number of Additional Modules Covered (Propagation Rate)	Number of Total Modules Covered (Cumulative Propagation)
1	2	2
2	4	6
3	6	12
4	8	20
5	10	.
6	12	.
7	14	.
.	.	.
.	.	.
.	.	.
n	2n	$n^2 + n$

Table 2.3 Propagation Statistics for Uni-directional Class D Network

The class D network is only 50% as fast as the class C network or the bi-directional network. A simple rearrangement of channels has made the resulting uni-directional network really inefficient. As was shown before the limit of total propagation in bi-directional to total propagation in uni-directional class C is one. The same limit for class D is:

$$\lim_{n \to \infty} \frac{T.P._{BD}}{T.P._{UD}} = \lim_{n \to \infty} \frac{2n^2 + 2n}{n^2 + n} = 2$$

The plots of cumulative propagation rates in the three types of networks and the limiting behaviors are depicted in Figure 2.8 and 2.9 respectively.

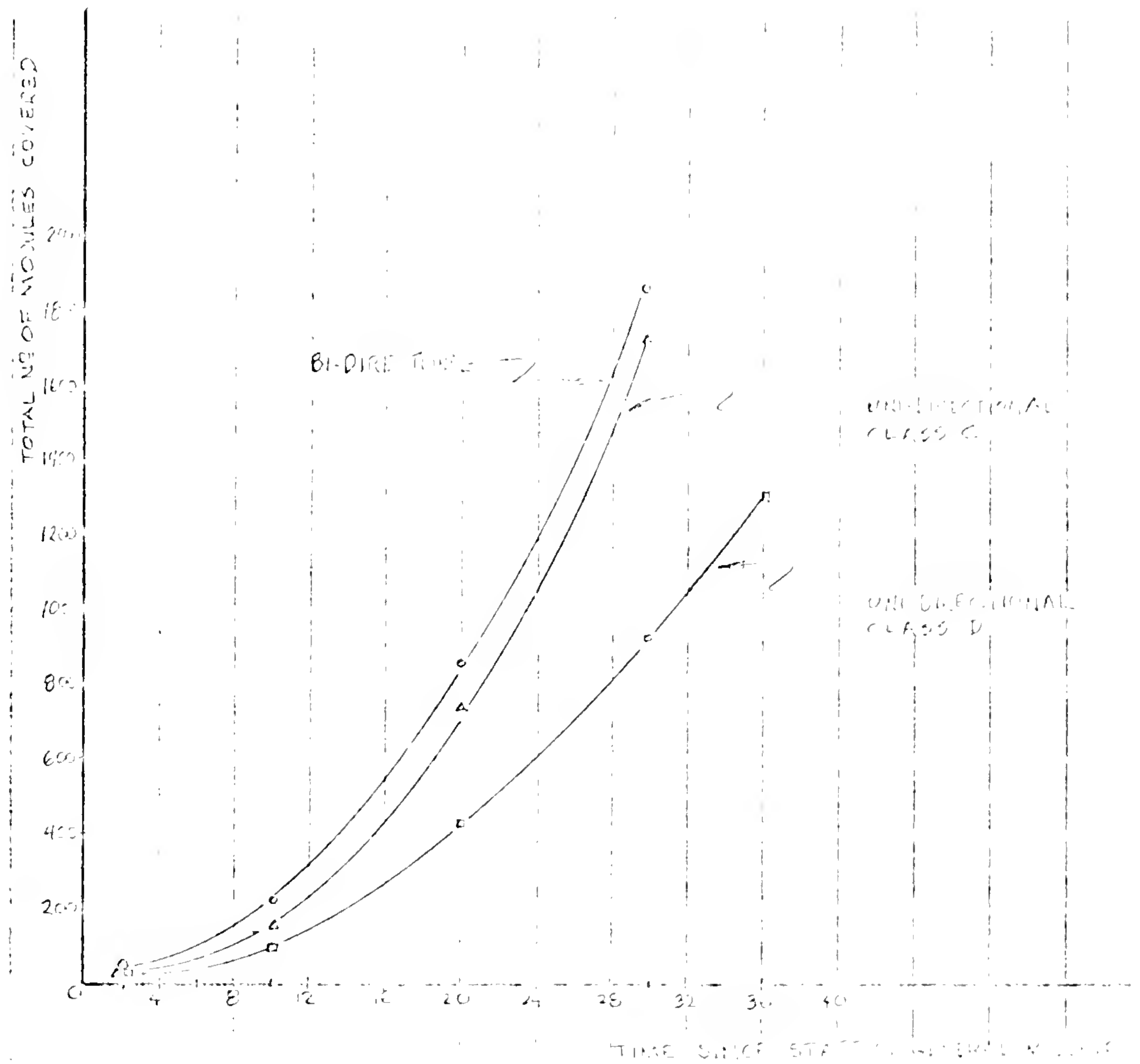


Figure 2.8 Cumulative Propagation Rates in Bi-directional Square, Uni-directional Square Class C, and Uni-directional Square Class D Networks.

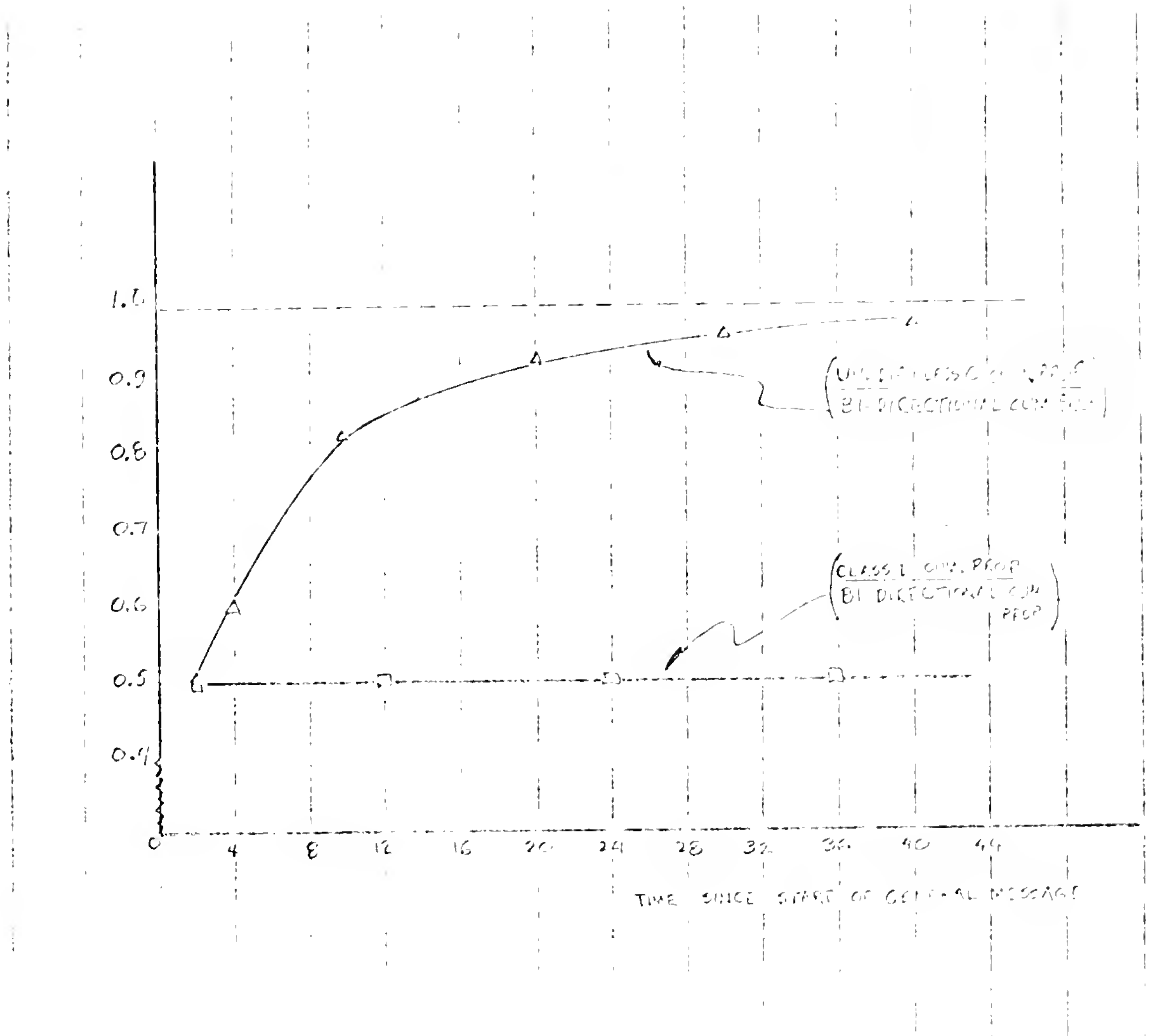
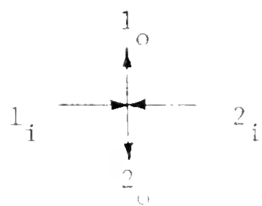


Figure 2.9 Ratio of Cumulative Propagation Rate in Bi-directional Square Network to Rates in Uni-directional Square Class C and Class D Networks.

Response Routing in the Class D Network

All possible transmission rules were empirically investigated and then the ones that resulted in smallest response times were chosen. As indicated before these rules were to be such that they relied only on the first arrival channel for the general message and could be applied in total ignorance about what other modules are doing. In the class D network this requirement cannot be entirely satisfied.

Consider the class D module. Let it be labeled clockwise i.e.



When the general message arrives on a single channel, a unique output channel can be defined so that the response, if and when it arrives, is transmitted on that channel. For instance, if the general message first arrival configuration, i, is (1,0) then we can choose j, the response transmission channel configuration, to be (0,1). This is unique in the sense that if the module is rotated by 90° or 180° or 270°, for the same input channel, the same output channel is specified.

However, if i = (1,1), that is if the general message arrives simultaneously on both channels, then we cannot find a unique j other than j = (1,1) which is ruled out for response transmission. For instance, if

following the convention in the diagram we say if $i = (1,1)$ then $j = (1,0)$, we could rotate the module by 180°, without altering i and end up with a different output channel. Empirical investigations verified this argument. It just was not possible to route the response solely on the basis of general message first arrival pattern. For routing the arrival channel configuration of the response had to be used in cases where $i = (1,1)$.

Let,

i = general message first arrival channel configuration

j = response transmission channel configuration

k = response arrival channel configuration

Then,

$i = (1,0) \rightarrow j = (0,1)$

$i = (0,1) \rightarrow j = (1,0)$

$i = (1,1)\ \&\ k = (1,0) \rightarrow j = (0,1)$

$i = (1,1)\ \&\ k = (0,1) \rightarrow j = (1,0)$

Notice that when $i = (1,1)$, if i is set equal to k then we simply follow the rules for $i = (1,0)$ or $(0,1)$.

Therefore we can write,

$i = (1,0) \rightarrow j = (0,1)$

$i = (0,1) \rightarrow j = (1,0)$

if $i = (1,1)$ set $i = k$

While still not complicated, these rules are more elaborate than the ones for the class C network. This makes class B module less efficient than the class C one.

The rules for the square class D network were applied to all the modules in a network of 240 modules. Response path patterns are as shown in Figure 2.10. The curly lines indicate the response paths. The solid lines are hypothetical "reflecting walls." Note the regions of turbulence.

The propagation and response delays were computed as for the class C network. The results appear in Figure 2.11. The solid lines connect modules having the same propagation and response delays. The divergence of delays is readily ascertained by examining the iso-delay lines.

On all counts (propagation efficiency, response efficiency, and logic and memory requirements) the class D network is considerably less efficient than class C. The major difference between class C module and the class D is that in the C type channels of the same kind (input or output) are adjacent whereas in class D the two types of channels alternate, i.e. . This difference may be responsible for the decreased efficiency in D. Chapter IV will examine this hypothesis.

Summary

In this section three types of networks were analyzed: 1) the bi-directional network having modules with two-way channels; 2) the uni-directional class C having modules with adjacent input and adjacent output channels, i.e. and 3) the uni-directional class D having modules with alternating input channels, i.e. .

A priori it was hypothesized that uni-directional networks would be slower in propagating a message than the bi-directional ones because of

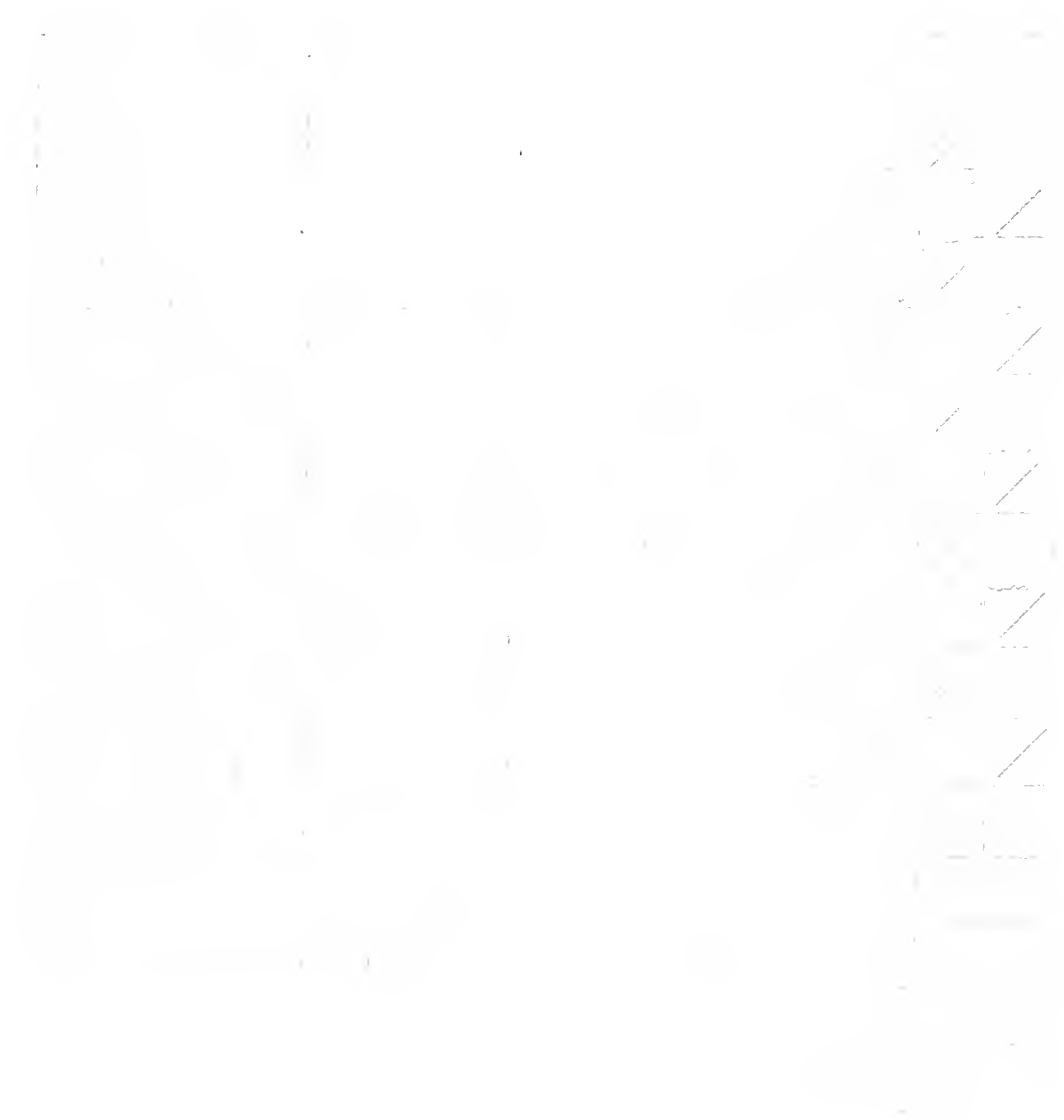


Figure 2.10 Response Return Paths in Uni-directional Square Class D Network.

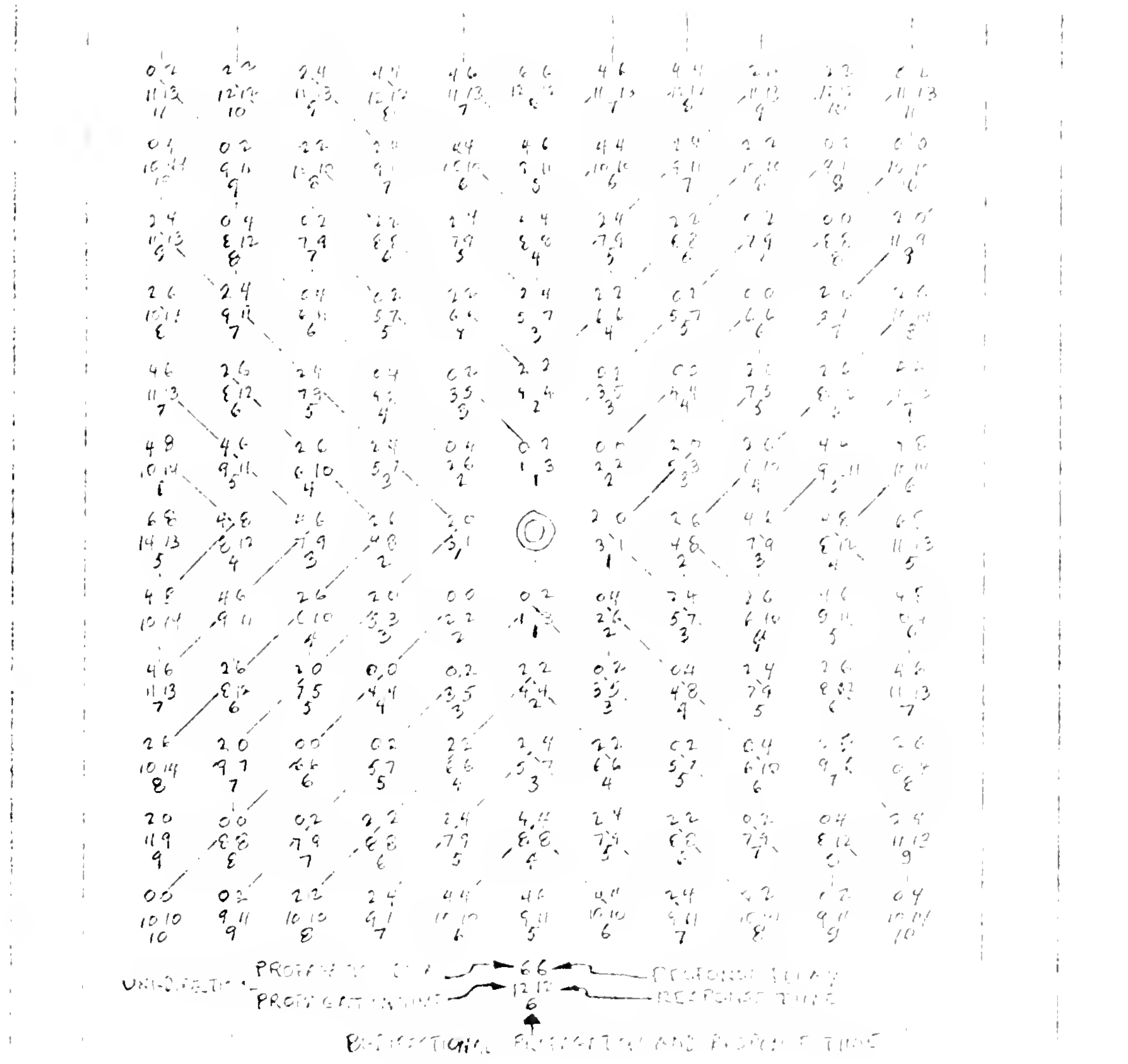


Figure 2.11 General Message Propagation and Response Return Times in Bi-directional Square and in Uni-directional Square Class D Networks.

the restrictions on flow directions. The hypothesis was rejected for class C. While class C is somewhat slower, its speed approaches that of bi-directional very rapidly. The hypothesis proved true for class D network. The non-adjacence of channels of the same orientation (i.e. either towards or away from the module) reduced propagation speed by about 50%.

It was shown that the bi-directional network solves our basic problem: allowing for routing of a response to the source of general message without addressing schemes. A priori it was hypothesized that the same would not be true in uni-directional networks because of constraints on propagation directions. It was shown that it is possible to define very simple rules such that response routing is achieved for class C networks. The uni-directional response times differed only a constant amount from bi-directional times.

The logic and memory requirements for the solution of the basic problem were analyzed. The uni-directional overall turned out to be superior in that it required <u>less</u> logic, less memory as well as less computation.

Response routing was investigated in the class D networks. This was shown to be possible. However, the difference between class D response time and bi-directional response time was shown to be increasing with the size of the network. Also more complicated routing rules are required.

It is therefore concluded that for the solution of the basic problem defined in this study, some but not all networks having one-way channels are superior to networks having two-way channels. This is shown to be true for square arrays. Its validity in hexagonal networks is yet to be demonstrated.

CHAPTER III

HEXAGONAL NETWORKS

Overview

This chapter will be devoted to studying which if any of the hexagonal unidirectional networks achieve propagation and response routing almost as rapidly as the hexagonal bi-directional networks. It will be shown that a mixed uni-directional arrangement compares favorably with hexagonal networks having two-way channels.

The Bi-directional Hexagonal Network

A hexagonal network was constructed as depicted in Figure 3.1. Each vertex represents a module. The ruled lines connect the module that receive the general message at the same time (isotemporal propagation). The same stopping rule used in the square case was used here i.e. the general message should not be propagated on channels that had brought it in. The propagation pattern is drawn on the network. Note the isotemporal propagation lines form hexagons. The propagation rates can be easily found by counting the number of modules touched by the isotemporal lines and noting the trends. The results appear in table 3.1.

Figure 3.1 Propagation of a General Message in Bi-directional Hexagonal Network.

Time	Number of Additional Modules Covered (Propagation Rate)	Number of Total Modules Covered (Cumulative Propagation)
1	6	6
2	12	18
3	18	36
4	24	60
.	.	.
.	.	.
.	.	.
n	6n	$3n^2 + 3n$

Table 3.1 Propagation in the Hexagonal Bi-directional Network

Compared to the square, the hexagonal arrangement has 50% more channels and the propagation rates are 50% greater.

The Uni-directional Hexagonal Class K Network

The class K module i.e. is very simple and symmetric; one channel points to the module, the adjacent one away from the module. Rotations of 120° or its multiple do not alter the module.

A network constructed using class K modules only appears in Figure 3.2. I have indicated the propagation pattern on the same diagram. Heavy lines represent iso-temporal propagation. The propagation rates determined by counting the modules associated with iso-temporal lines appear in table 3.2.

Figure 3.2 Propagation of a General Message in Unidirectional Hexagonal Class K Network.

Time	Number of Additional Modules Covered (Propagation Rate)	Number of Total Modules Covered (Cumulative Propagation)
1	3	3
2	6	9
3	3	18
4	12	30
.	.	.
.	.	.
.	.	.
n	3n	$\frac{3}{2}n^2 + \frac{3}{2}n$

Table 3.2 Propagation in Hexagonal Class K Network

The relative invariance of class K to rotation and its symmetry had lead us to expect the hexagonal module to have propagation rates comparable to the bi-directional one. It is less efficient than even the square class C.

Response Routing in the Uni-directional Hexagonal Class K Network

Is it possible in the hexagonal networks, too, to specify simple response routing rules based solely on what channel(s) the general message arrived first? These rules are to be same for all modules of the same class. When each module applies the rule (in total ignorance of where other modules are located and what they are doing) to route a response, the response is to reach the source of the general message. To repeat again each module will operate independently and without knowing where the module that generated the general message and the module that is sending the response is located.

Such rules exist for hexagonal class K as they did for square class C and for class D.

Let the channels be labeled clockwise and let:

i = general message first arrival channel configuration

j = response transmission channel configuration

Restated the question is:

is $j = f(i)$?

The answer is yes, the function being as follows:

$j = (1,0,0)$ if $i = (0,1,0)$ or $i = (1,1,0)$

$j = (0,1,0)$ if $i = (0,0,1)$ or $i = (0,1,1)$

$j = (0,0,1)$ if $i = (1,0,0)$ or $i = (1,0,1)$

The matrix formulation is:

for single arrivals

$$j = Si \qquad S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

for multiple arrivals

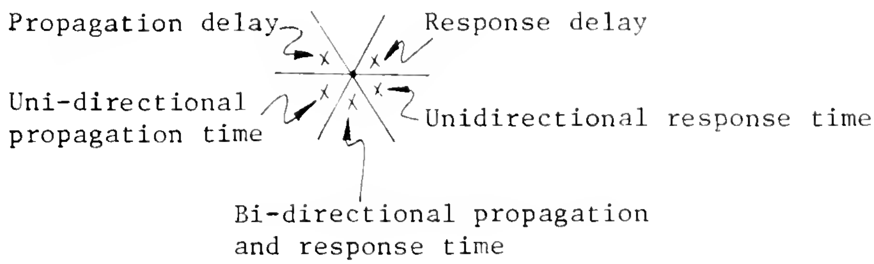
$$j = Mi \qquad M = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

These rules were applied to all the modules in the network. The resulting response pattern appears in Figure 3.3. Notice the three regions of turbulence at 120° degrees to each other.

The time it takes a general message to reach a node and the time it would take a response to reach the request source both in the uni-directional

Figure 3.3 Response Return Paths in Uni-directional Class K Network.

hexagonal class K and in the bi-directional networks were computed. The results for each module are displayed in Figure 3.4 in the manner shown below:



An examination of the iso-delay lines reveals that both the propagation and the response iso-delay lines diverge. Evidently hexagonal class K lacks propagation and response efficiency. This class is analogous to square class D since in both modules input and output channels alternate with each other. It will be recalled that in the square networks, too, alternating channel configurations proved to be slow while the adjacent-channel arrangements of class C i.e. were almost as rapid as the bi-directional one. The hexagonal analog of square class C is hexagonal class D i.e. . Therefore we would expect the hexagonal class D network to be a rapid one.

The Uni-directional Hexagonal Class D Network

It is not possible to construct a network of class D modules with connectedness in all regions. The networks of class D type divide themselves into two regions each one being disconnected in the sense that a

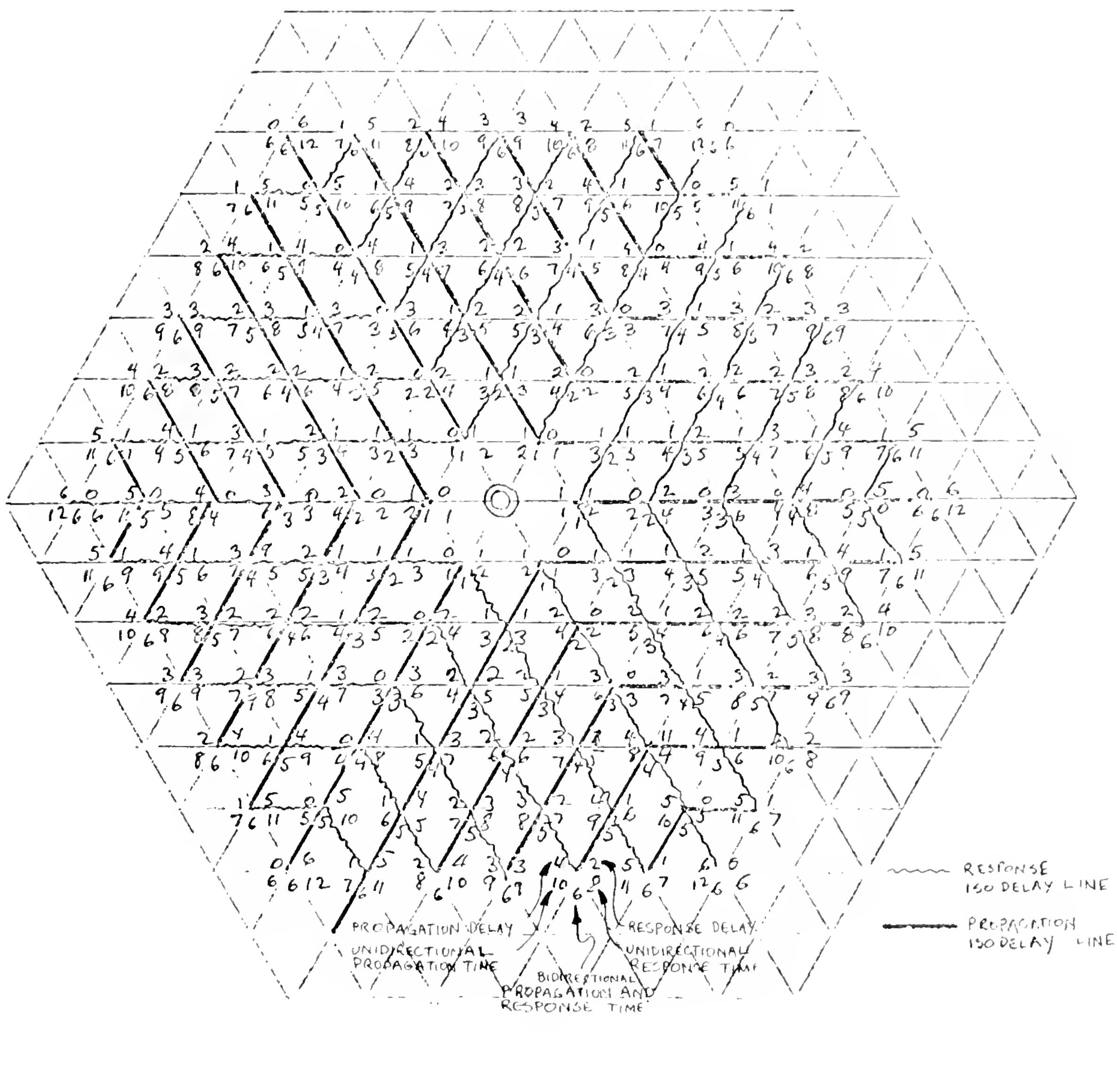


Figure 3.4 General Message Propagation and Response Return Times and Delays in Uni-directional Hexagonal Class K Network. Solid Lines are Iso-delay Lines.

path does not always exist between any pair of modules. This is easily seen by an examination of the module. The input and output channels form three directions of flow. Since the output of one is the input of the next module, the same three directions continue. Hence flow in the opposite three directions becomes impossible.

As was stated a little earlier, the square analog of the hexagonal class D is square class C. The network constructed with square class C allows straight flow in four directions via alternating lines. In the hexagon, too, a network was constructed such that flow in all six directions via alternating lines became possible. The result was a mixed graph containing three class D modules for each class K one. The connected mixed graph thus obtained was used as an approximation of the unsuitable hexagonal class D network.

The hexagonal D+K network is drawn in Figure 3.5 and the propagation pattern of a general message emanating from the center of the graph is shown therein. Because the pattern is complicated, a depiction of only the pattern appears in figure 3.6.

The propagation rates can again be determined by counting the modules associated with the iso-temporal propagation lines. The results are shown in table 3.3.

Figure 3.5 Propagation of a General Message in Unidirectional Hexagonal K+D Network.

Figure 3.6 General Message Propagation Pattern in Uni-directional Hexagonal Class K+D Network.

Time	Number of Additional Modules Covered (Propagation Rate)	Total Number of Modules Covered (Cumulative Propagation)	
		Uni-dir.Hex. Class D+K	Bi-dir.Hex.
1	3	3	6
2	9	12	18
3	15	27	36
4	27	54	60
5	21	75	90
6	45	120	126
7	27	147	168
8	63	210	216
9	33	243	270
10	81	324	330

Table 3.3 Propagation in Uni-directional Hexagonal Class D+K.

The propagation rate appears highly irregular. To find some order, the data were plotted. The graph is in Figure 3.7. Also plotted are the propagation rates for the bi-directional hexagonal network and the uni-directional class K network.

The rate fluctuates between two envelopes and crisscrosses the hexagonal bi-directional line. The lower envelope is parallel to the class K line. By symmetry the upper envelope should be parallel to class D propagation rate line. This suggests that if a connected hexagonal class D network could be constructed it would be faster than the bi-directional line since the slope of the upper envelope is greater than the slope of the bi-directional line.

From only the propagation rate it is not easily possible to determine the speed of the uni-directional class D+K with respect to the bi-directional. Therefore, I plotted the cumulative propagation, and also included the bi-directional cumulative propagation data in table 3.3. The plot is in Figure 3.8.

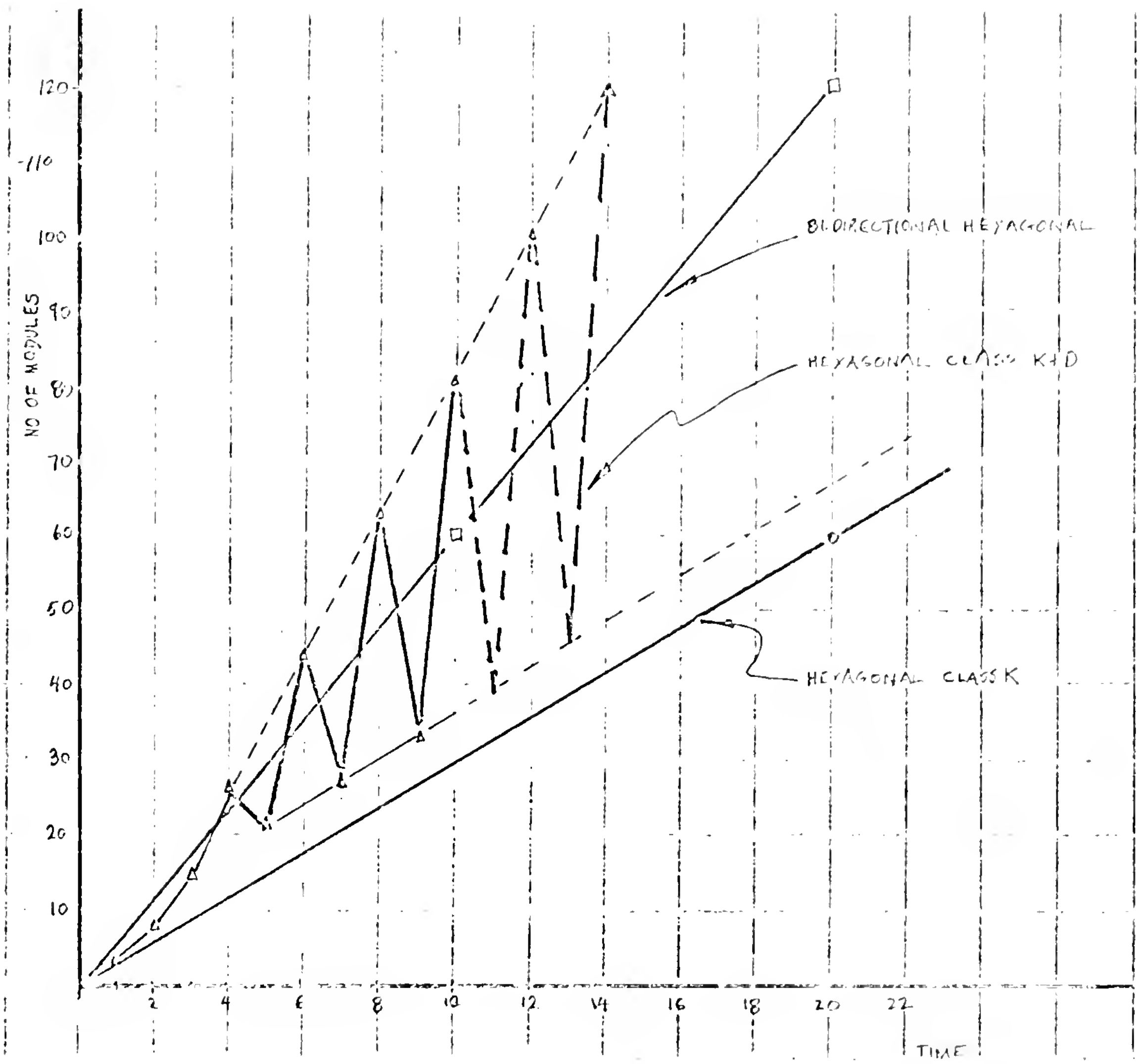


Figure 3.7 Propagation Rates in Bi-directional Hexagonal Network and in Uni-directional Hexagonal Class K and Class K+D Networks.

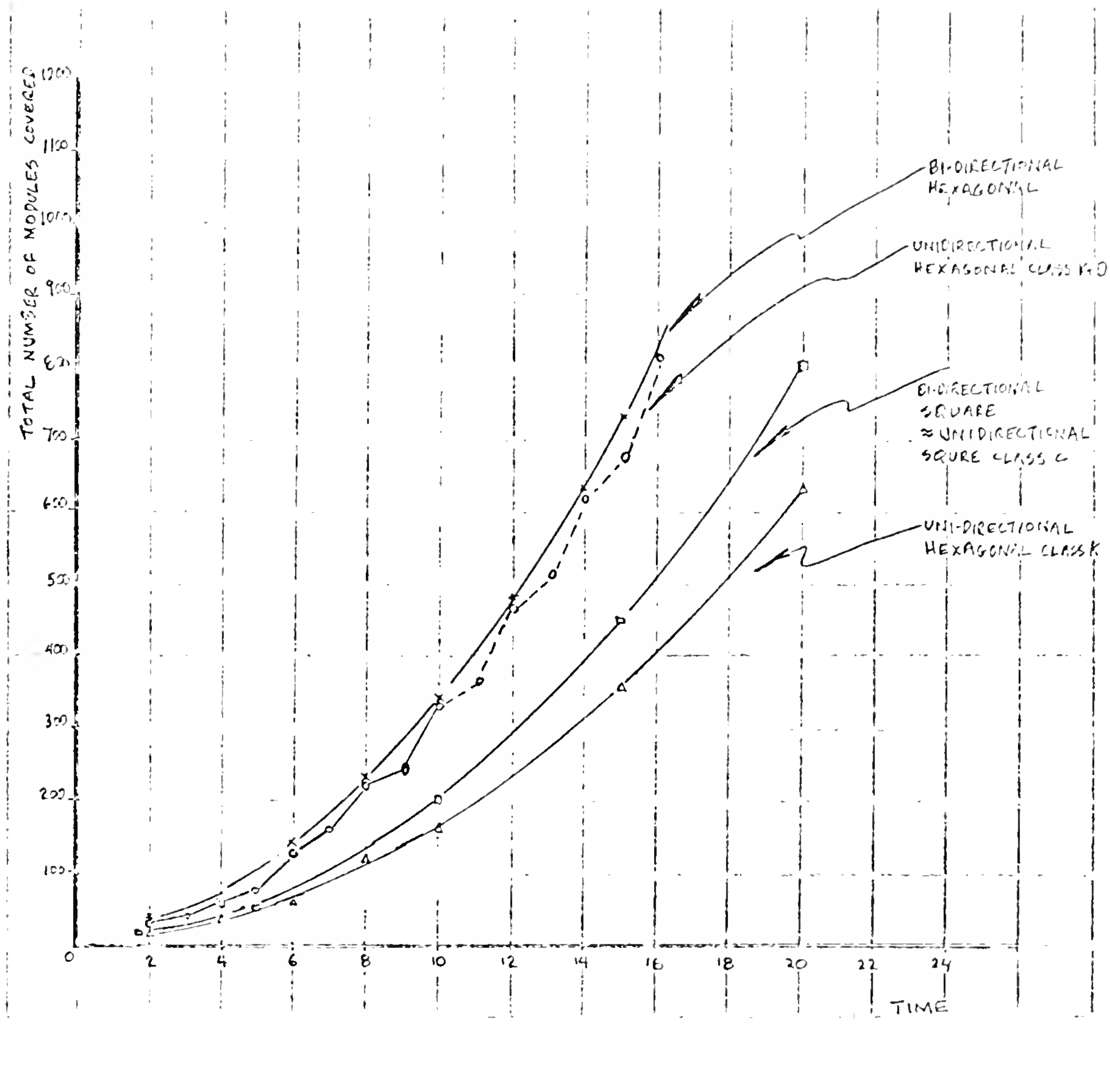


Figure 3.8 Cumulative Propagation in Bi-directional Square, Bi-directional Hexagonal, Uni-directional Hexagonal Class K and in Uni-directional Hexagonal Class K+D Networks.

The hexagonal class K+D line follows closely the hexagonal bi-directional cumulative line. An examination of table 3.3 reveals that at even times there is a constant difference of six between the cumulative propagations. Notice how inefficient the class K network is.

Once again, the inclusion of modules having adjacent channels of the same orientation has made uni-directional almost as rapid as the bi-directional.

Response Routing in the Uni-directional Hexagonal Class K+D Network

Following the clockwise labeling convention and the notation used in class K discussions, the rules shown below apply to a hexagonal Class D module:

$j = (0,0,1)$ if $i = (1,0,0)$ or $i = (1,1,0)$

$j = (1,0,0)$ if $i = (0,0,1)$ or $i = (0,1,1)$

$j = (0,1,0)$ if $i = (0,1,0)$ or $i = (1,0,1)$

Matrix rules:

$$j = Si \qquad S = \begin{bmatrix} 001 \\ 010 \\ 100 \end{bmatrix}$$

Multiple arrivals

$$j = Mi \qquad M = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Application of these rules to each module in the network resulted in the response transmission pattern in Figure 3.9. Since some of the modules are class K type, for them the class K rules were used.

In as much as class K rules were derived from a pure class K network it was not clear or expected that they would apply to the mixed D+K network. That together with class D rules they indeed result in fast response times might be a special case. Further research is needed to determine the extent of context-independence of rules for a module of a particular class.

The regions of turbulence that existed in the class K network response pattern are non-existent for the class D+K case. Therefore D+K appears faster. To determine just how much faster, propagation and response delays were computed as was done for class K. Figure 3.10 displays the results. The propagation and response statistics for both the uni-directional hexagonal K+D network and for the bi-directional hexagonal network are shown for each module. The solid lines show either iso-delay propagation or iso-delay response. For clarity, iso-delay lines are redrawn in Figures 3.11 and 3.12. An inspection of the iso-delay graphs reveals that both the propagation and response delays remain stable. In fact the following overall statistics can be computed.

Figure 3.9 Response Return Paths in Uni-directional Hexagonal Class K+D Network.

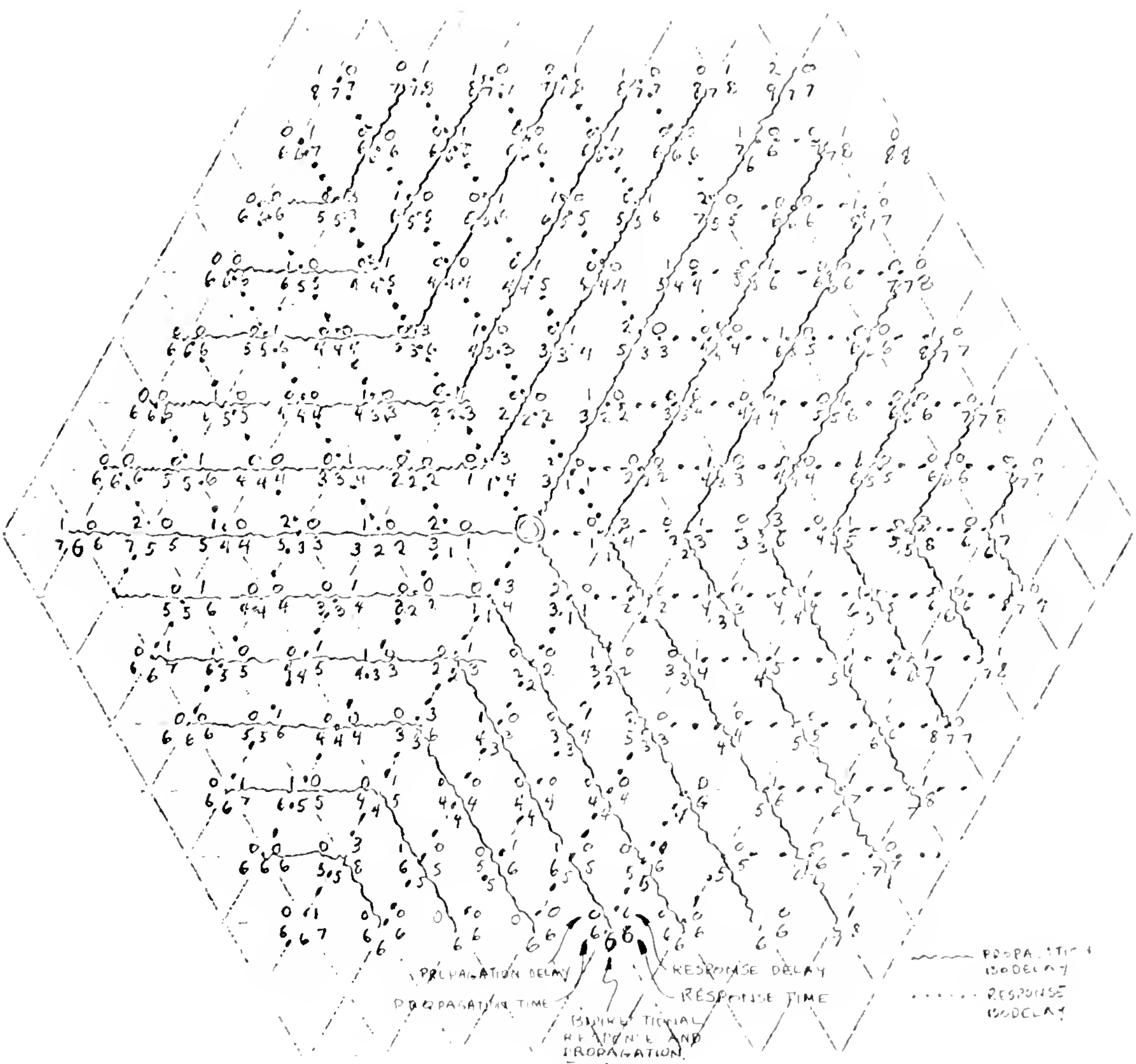


Figure 3.10 General Message Propagation and Response Return Times and Delays in Uni-directional Hexagonal Class K+D Network.

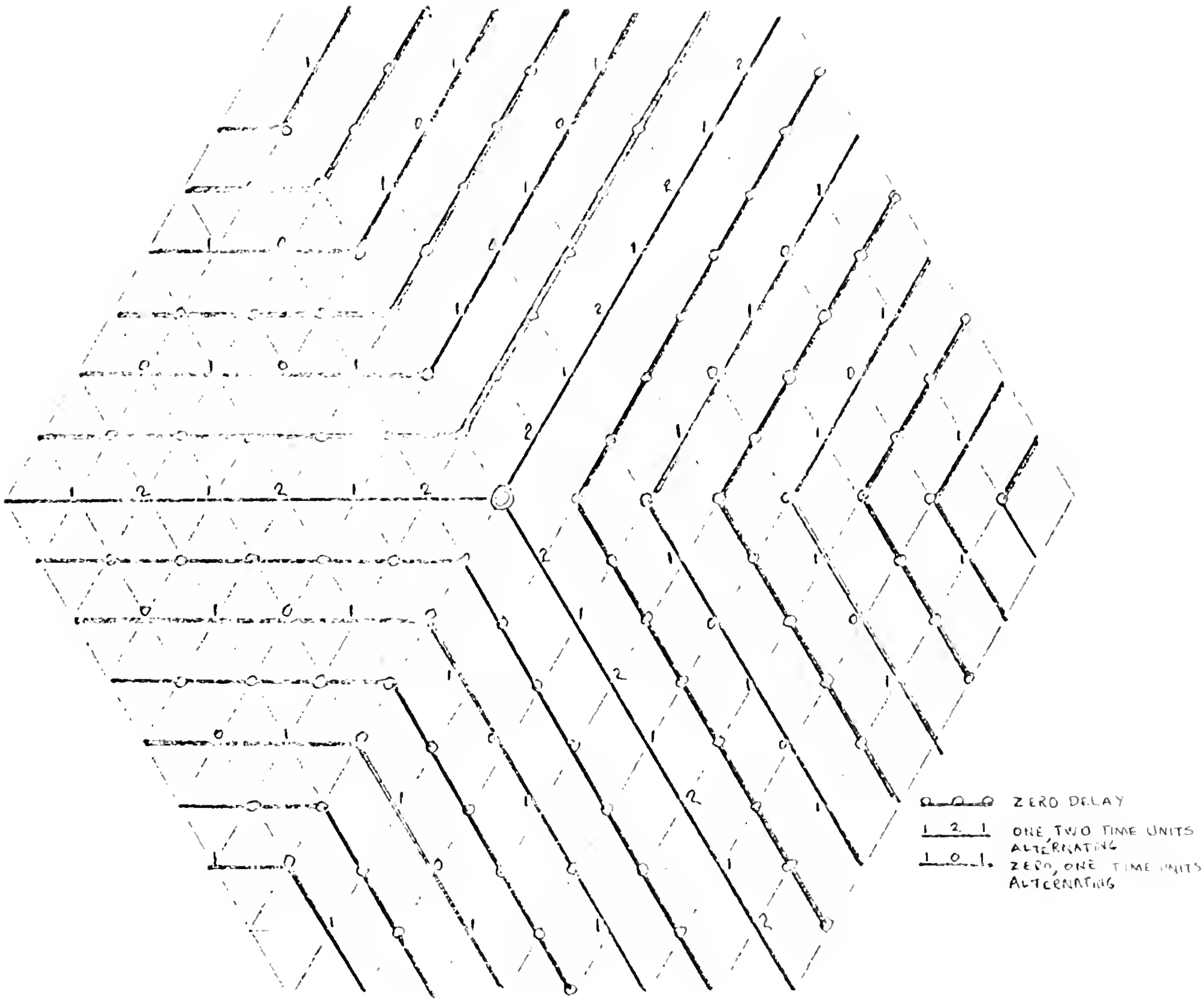


Figure 3.11 Uni-directional Hexagonal Class K+D Network Propagation Isodelay lines.

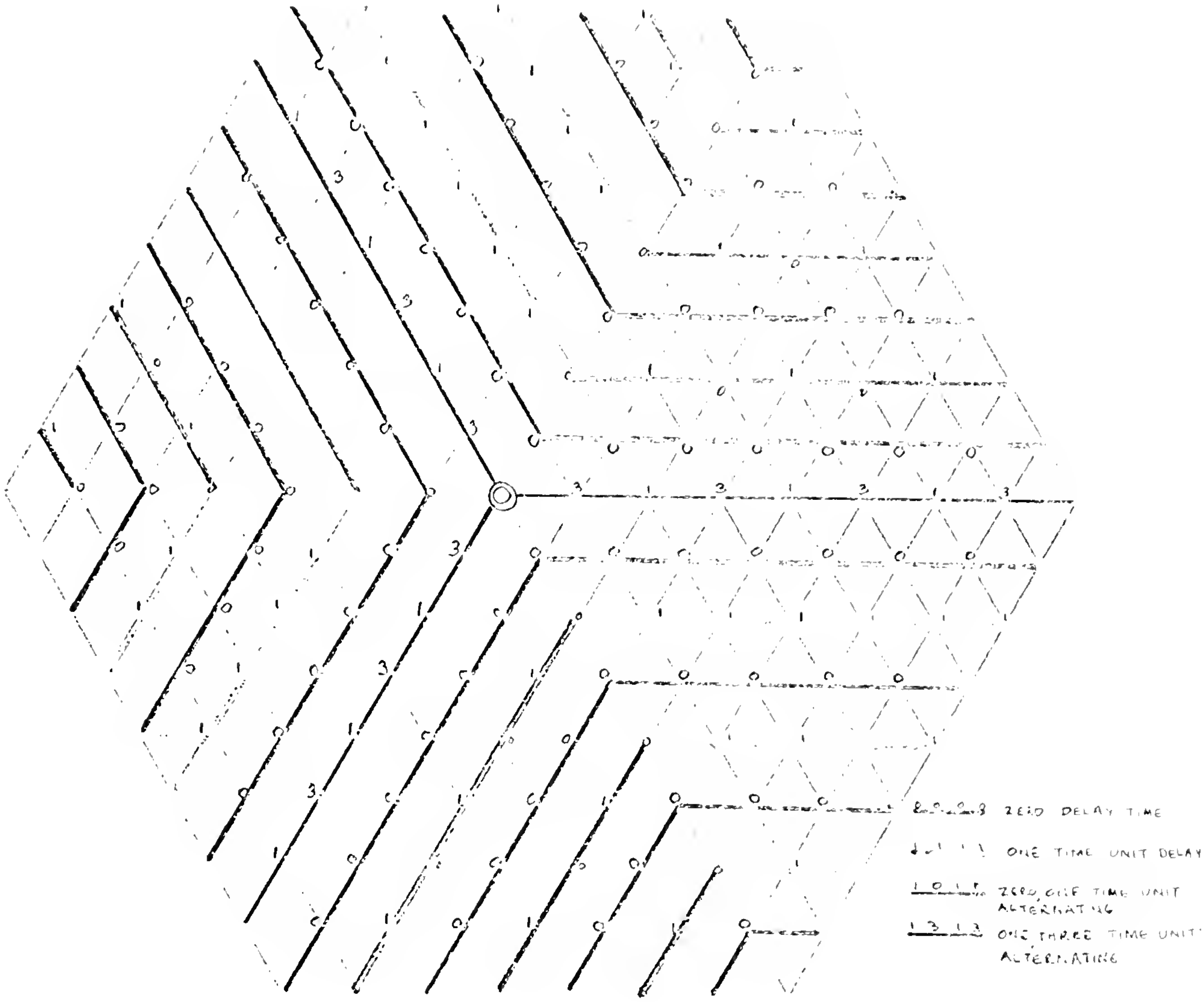


Figure 3.12 Uni-directional Hexagonal Class K+D Network Response Isodelay Lines.

Propagation

Maximum delay = 2 time units
Minimum delay = 0
Average delay = $\frac{1}{4}$

Response

Maximum delay = 3
Minimum delay = 0
Average delay = $\frac{3}{8}$

Overall average delay

$$\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$$

Both from propagation formulas and delay figures the efficiency of uni-directional hexagonal class K+D becomes obvious. The results tend to support the hypothesis that adjacence of the channels having the same orientation is a better arrangement. The next chapter will dwell on this as well as an alternative hypothesis.

At this point, the hexagonal K+D can be contrasted with square class D. The hexagonal K+D is 50% faster. However, it requires 50% more channels and 50% more rules. Each rule contains 50% more bits. The preferability of one over the other would depend on the value of time compared to costs of channels, logic, and memory.

Summary

In this section hexagonal bi-directional networks were contrasted with uni-directional hexagonal K and hexagonal D +K networks. Class K proved to be very slow, in fact slower than the square class C

whereas hexagonal D+K was almost as fast as the hexagonal bi-directional network. While hexagonal K network has propagation and response delays that increase with the size of the network, the hexagonal class D+K network has an average propagation-plus-response delay of about one time unit and this does not change with the size of the network.

These findings suggest that adjacence of channels of like direction increases propagation and response speed and that some uni-directional networks can be as rapid as bi-directional ones while requiring less logic and memory.

CHAPTER IV

POSSIBLE DETERMINANTS OF EFFICIENCY IN UNI-DIRECTIONAL NETWORKS

Overview

Two factors are identified as influencing the speed of propagation (considered to be an index of efficiency) in the networks with one-way channels: 1) the adjacence of channels with the same direction and 2) the number of directions in which flow is possible in straight lines. The two factors are initially considered to be alternate hypotheses. Networks are constructed fulfilling the premises of each hypothesis to various degrees. Actual results are compared with the predictions from each hypothesis. It is concluded that both factors are important, adjacence being more so.

Possible Determinants of Efficiency

While the results of the previous chapters indicate that adjacence of channels with the same orientation increases speed, it is not clear that this is the only relevant factor. An examination of both the square class C network and the hexagonal class K+D network reveals the following: straight flow is allowed in all directions. Along a straight line channels are all pointed in the same direction. Below is a small piece of the square class C network to illustrate this point.

Notice the directions of the lines of flow alternate in direction.

Therefore, a second hypothesis is that uni-directional network efficiency is related to the extent of straight flow provided. The "measure of efficiency" to be used here is the propagation rate (speed) only; if propagation is slow so would the response be. If propagation and response are slow clearly delays would be divergent.

It is not clear that the dependence of speed on the extent of straight flow is an alternate hypothesis. There is no way of ruling out the possibility that both factors are important or interdependent. However, I will create a straw man by considering the above to be an alternate hypothesis. To test the two hypotheses I will examine the following networks each fulfilling the premises of one or the other hypothesis or both, as indicated. For each network the predictions by each hypothesis are also included.

1. Hexagonal class I

The hexagonal class I module is intermediary between class D in which all incoming channels are adjacent as well as all outgoing channels, and class K which is at the other extreme. The network constructed with class I modules does not allow straight flow in any direction. Therefore if the straight-flow hypothesis is correct, class I network should be less efficient in propagation than class K network which allows straight flow in three directions. If the adjacence hypothesis is correct, class I should be more efficient than class K network.

2. Hexagonal I+D (3 class I modules for each of class K).

This mixed network has more adjacent channel arrangement than pure I network. Straight flow is provided by every other line in three directions only. Adjacence hypothesis would predict that this network is more efficient than pure I network. Straight-flow would predict that while this would be more efficient than pure I which allows no straight flow, that it would be less efficient than class K since class K network provides straight flow in three directions but has more flow lines in each direction.

3. Hexagonal class I+D+K (12 of I, 3 of D for each of K).

This network provides straight flow by every other line in all six directions. According to the adjacence hypothesis I+D+K network should be less efficient than I+D network while straight-flow would place it ahead of pure I networks which provide no straight flows.

4. Hexagonal class D+K; Type I.

This was studied in the previous chapter. It provides straight flow in all six directions and has mostly adjacent iso-direction channel modules. Therefore both hypotheses would place it at the top of the efficiency list.

5. Hexagonal class D+K; Type II.

While in this network the ratio of D to K remains the same and straight flow is provided in all directions, two adjacent lines point in one direction and for the next two lines the opposite direction prevails. Adjacence hypothesis would predict no difference in efficiency with respect to D+K Type I while the alternate hypothesis would.

6. Hexagonal class K.

This type of network was also studied in the previous chapter. It has no adjacence of iso-direction channels and has straight flow in three directions at 120° to each other. The first hypothesis would put this at the low end of efficiency scale while the second one would assign an intermediary position.

7. Hexagonal mixed.

This network has modules of classes C, J, K, L, D. Straight flow is not provided for. Therefore while the first hypothesis would place it at an intermediary position, the alternate hypothesis would place it at the bottom.

In summary, the following efficiency rankings would be predicted:

Back-to-front channel orientation hypothesis:

(D+K Type I) = (D+K Type II) > (I+D) > (I+D+K) > I > Mixed > K

Straight-flow hypothesis:

(D+K Type I) > (D+K Type II) > K > (I+D+K) > (I+D) > I = Mixed

Construction

Construction of class I, class I+D, and class I+D+K are shown in Figures 4.1 and 4.2. Class D+K Type II was constructed by pointing two adjacent lines in one direction and then the next two adjacent lines in the opposite direction. Class K and class D+K Type I were described in the previous chapter.

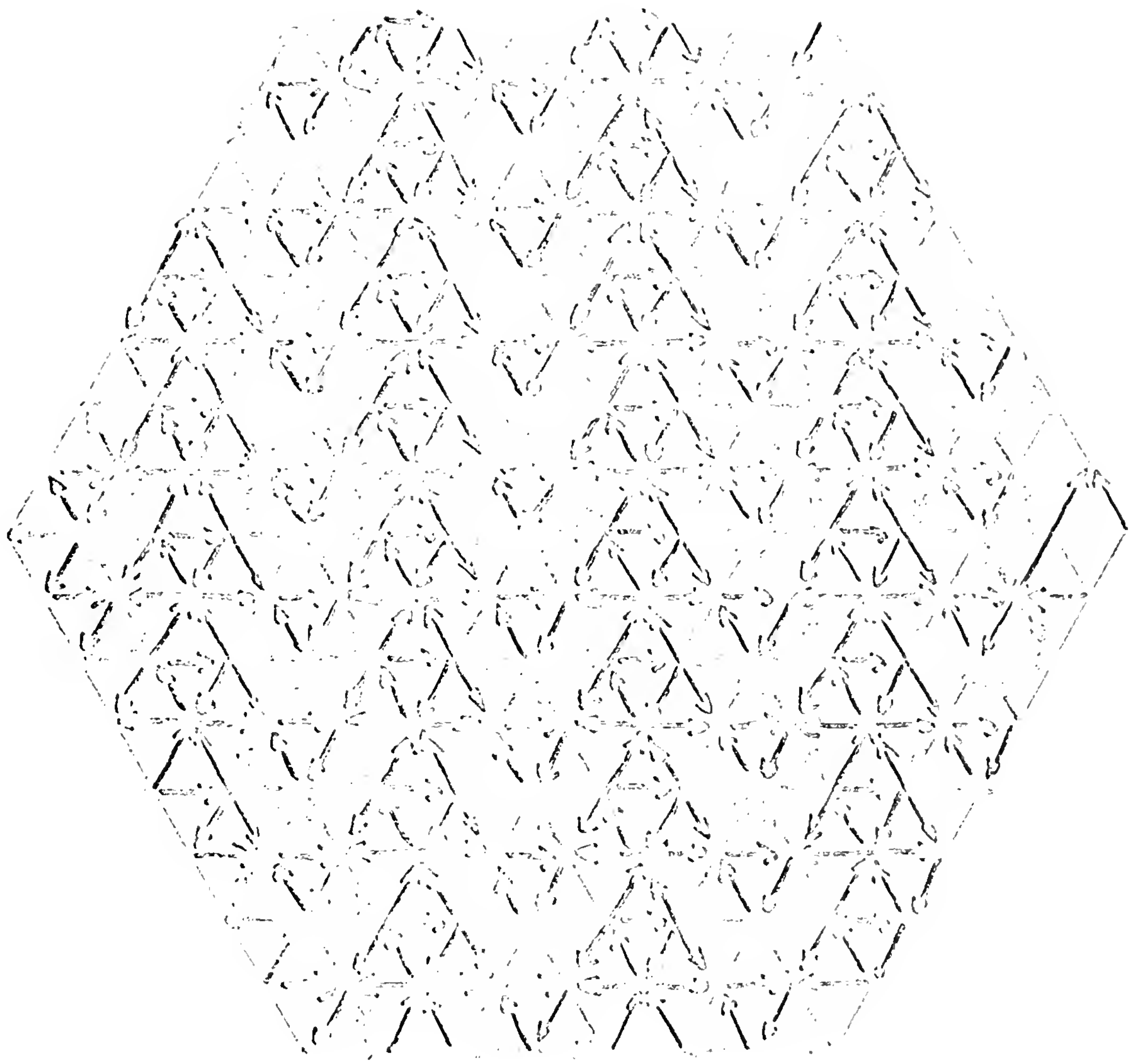


Figure 4.1 Construction of Uni-directional Hexagonal Class I Network.

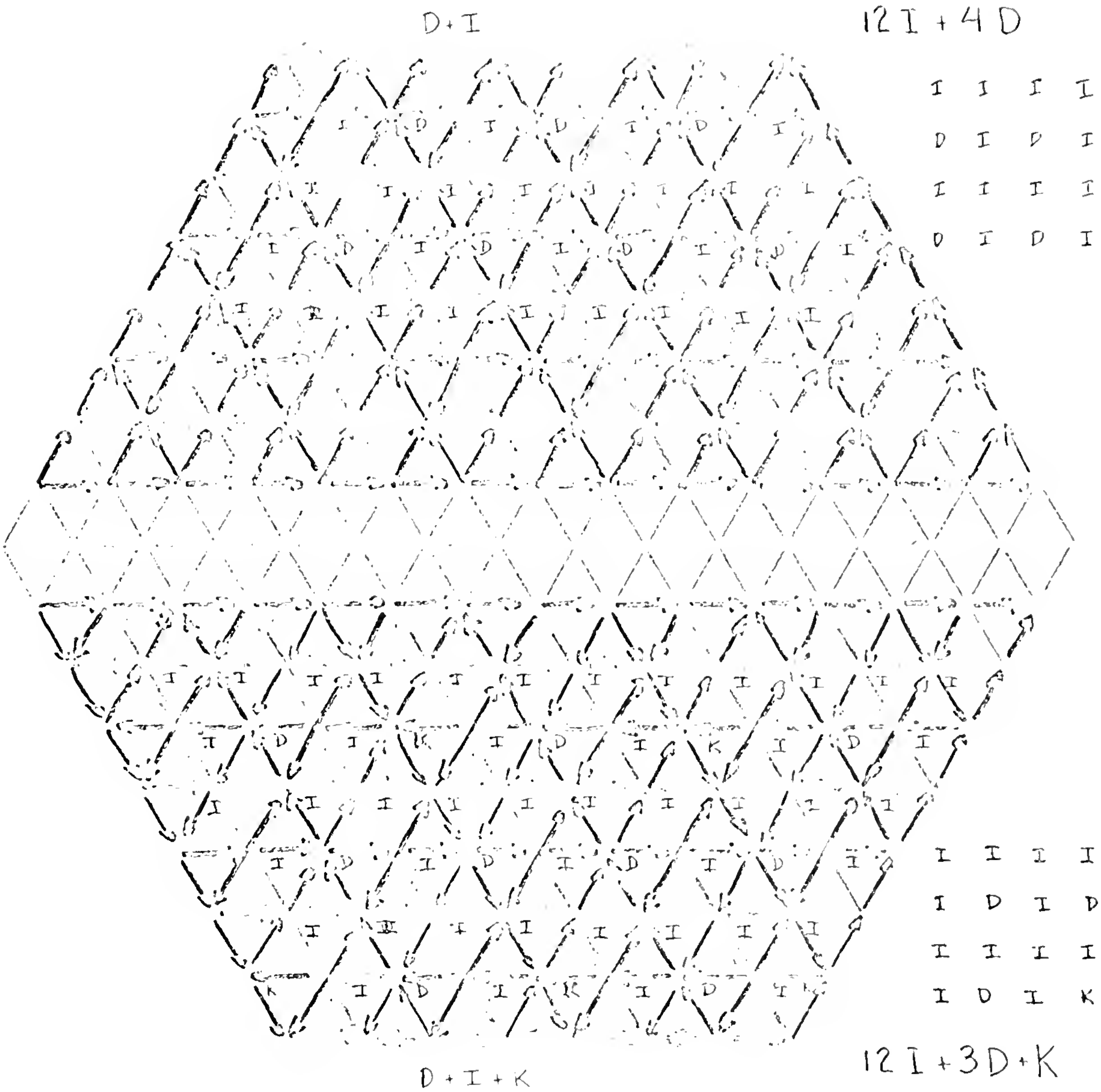


Figure 4.2 Constructions of Uni-directional Hexagonal Class D+I and Class D+I+K Networks.

Propagation Behaviors

Propagation of general messages in the various networks are depicted by Figures 4.3 through 4.7 the propagation rates can be determined by counting the number of additional modules covered at each time increment. The results appear in Table 4.1. Figure 4.8 contains the plots of propagation rates. Notice how the lines criss-cross. Because of the criss-crossing, the efficiency ranking is not easily determinable. Therefore in 4.9 a plot of cumulative propagations is shown. Now ranking is at once obvious. In fact we have the following.

(D+K Type I) > (D+K Type II) > (I+D+K) > Mixed > I > (I+D) > K

This result was not predicted by either of the hypotheses, though portions are explainable by one or the other hypothesis. It appears that efficiency is a function of both factors, i.e. straight flow and adjacence of channels having the same orientation. Perhaps adjacence is a more important factor because considering it only results in a better prediction than considering the other only.

In the prediction made by the channel orientation hypothesis, replacing I+D by I+D+K and then substituting Mixed for I+D and I+D for Mixed and changing the first "=" to ">" yields a prediction identical with the empirical result. The straight flow-prediction contains more errors since I+D and I must be interchanged, Mixed and K must be interchanged and then Mixed and I+D+K must be interchanged.

It is entirely possible that there are other factors at work whose identification would greatly improve our ability to predict the efficiency

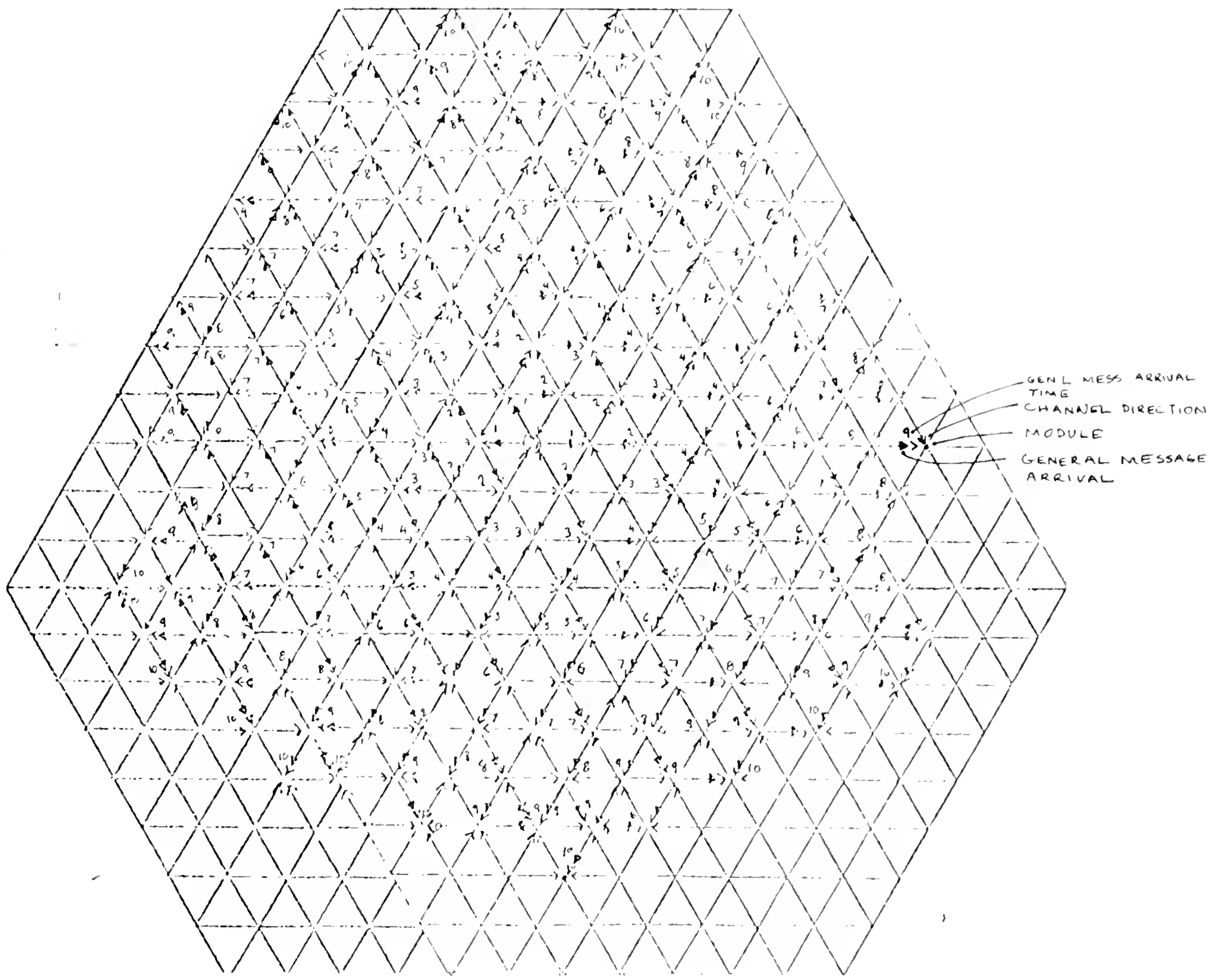


Figure 4.3 General Message Propagation in Uni-directional Hexagonal Class I Network.

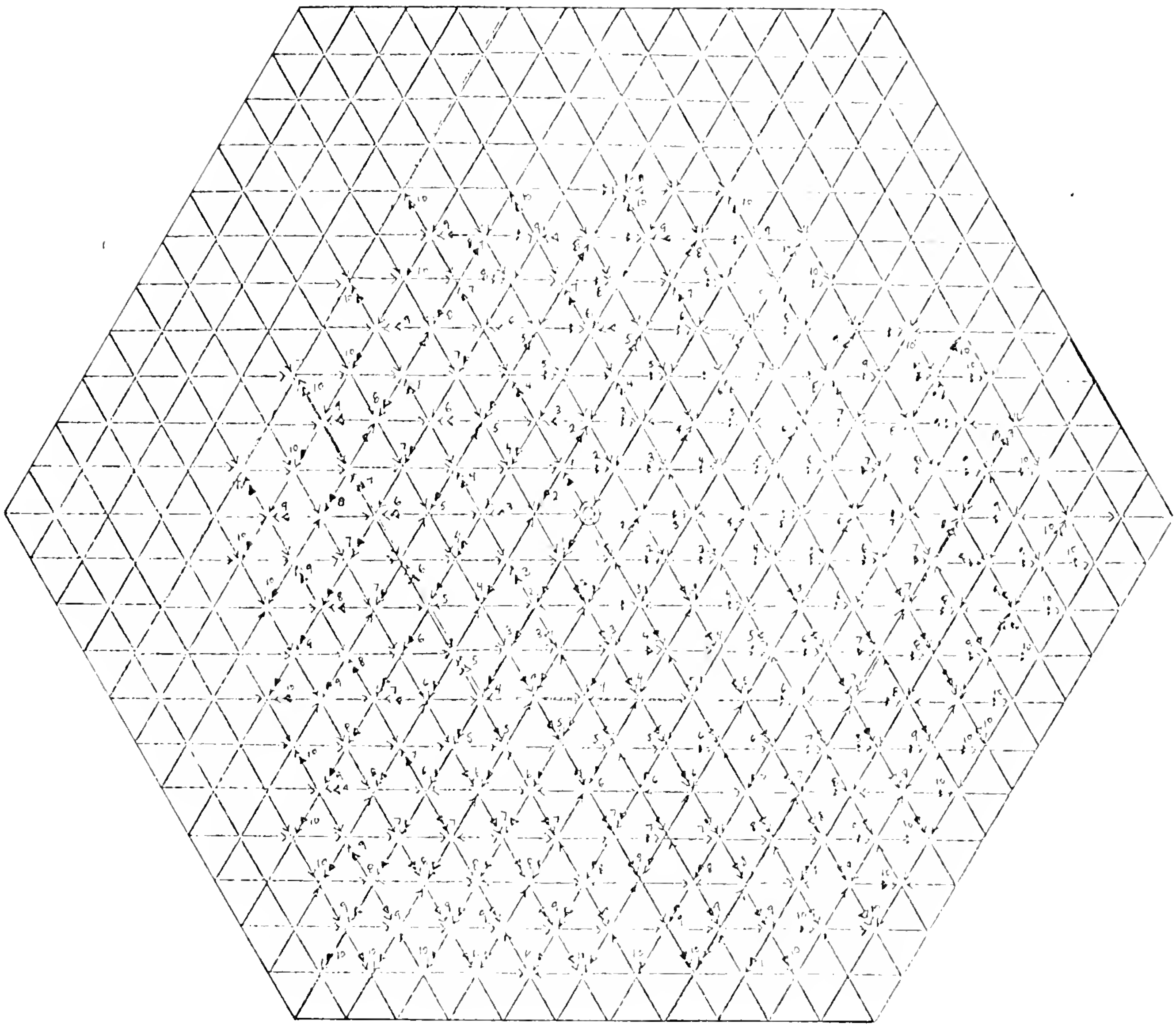


Figure 4.4 General Message Propagation in Uni-directional Hexagonal Class D+I Network.

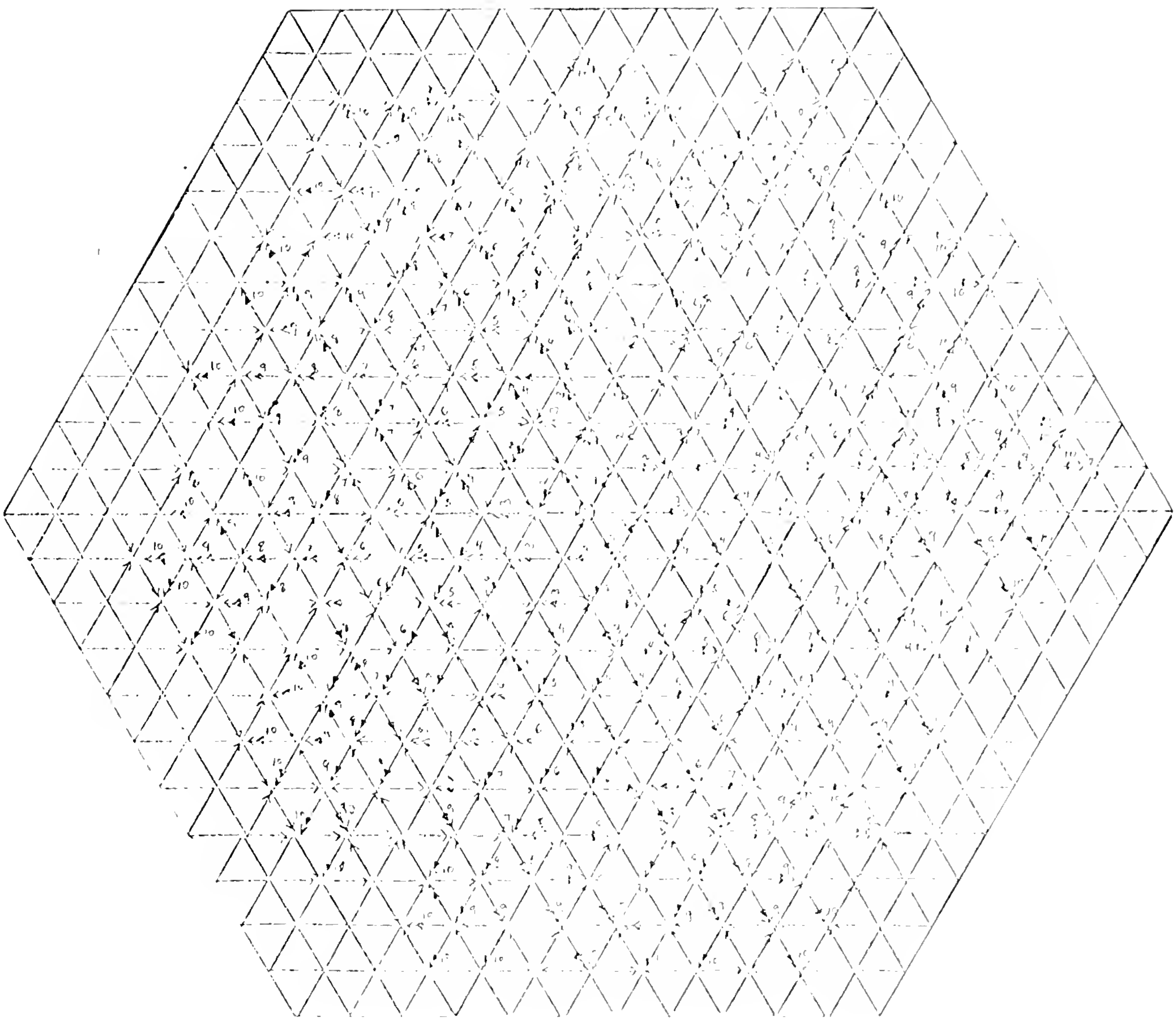


Figure 4.5 General Message Propagation in Uni-directional Hexagonal Class D+I+K Network.

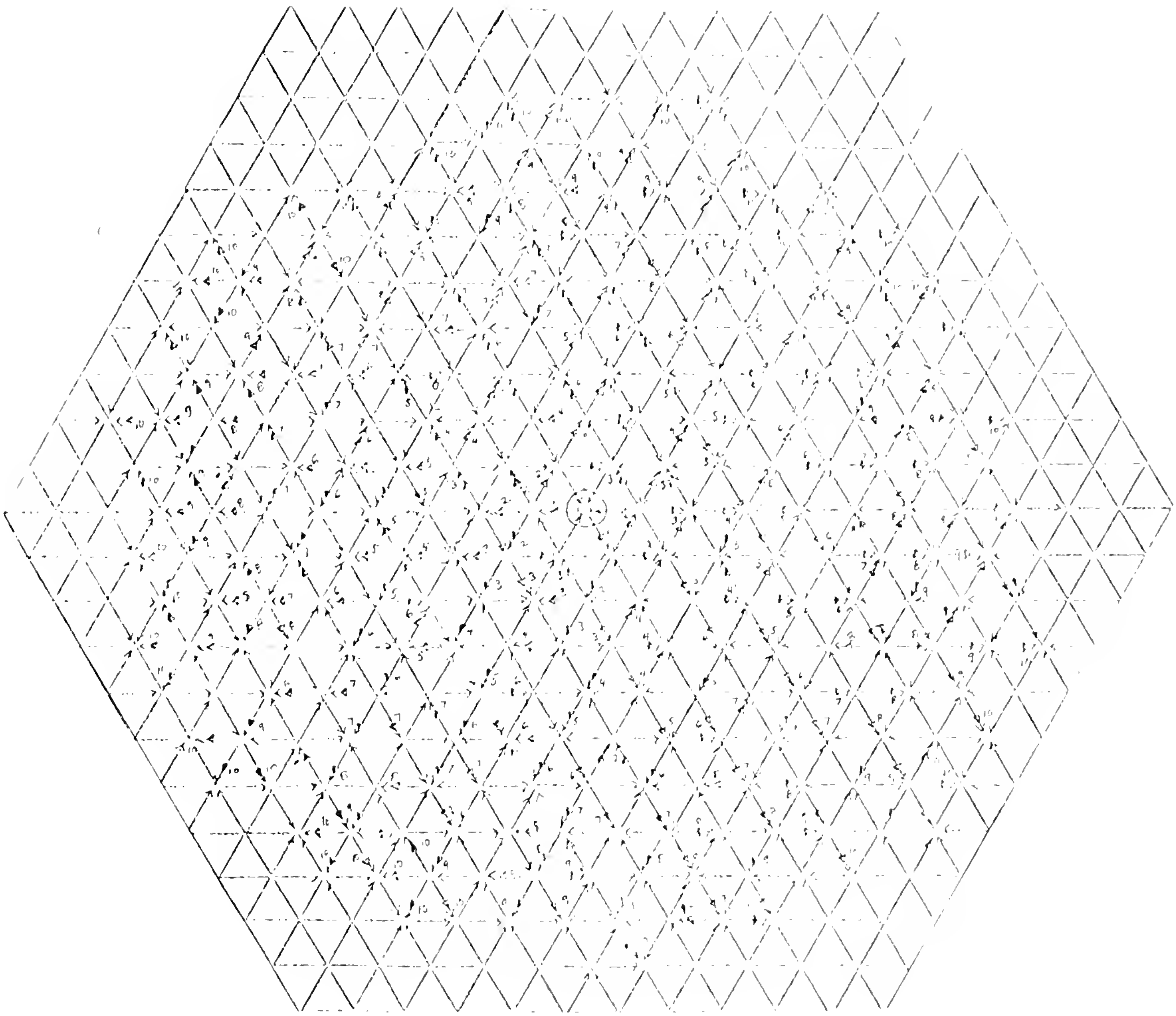


Figure 4.6 General Message Propagation in Uni-directional Hexagonal Mixed Class Network.

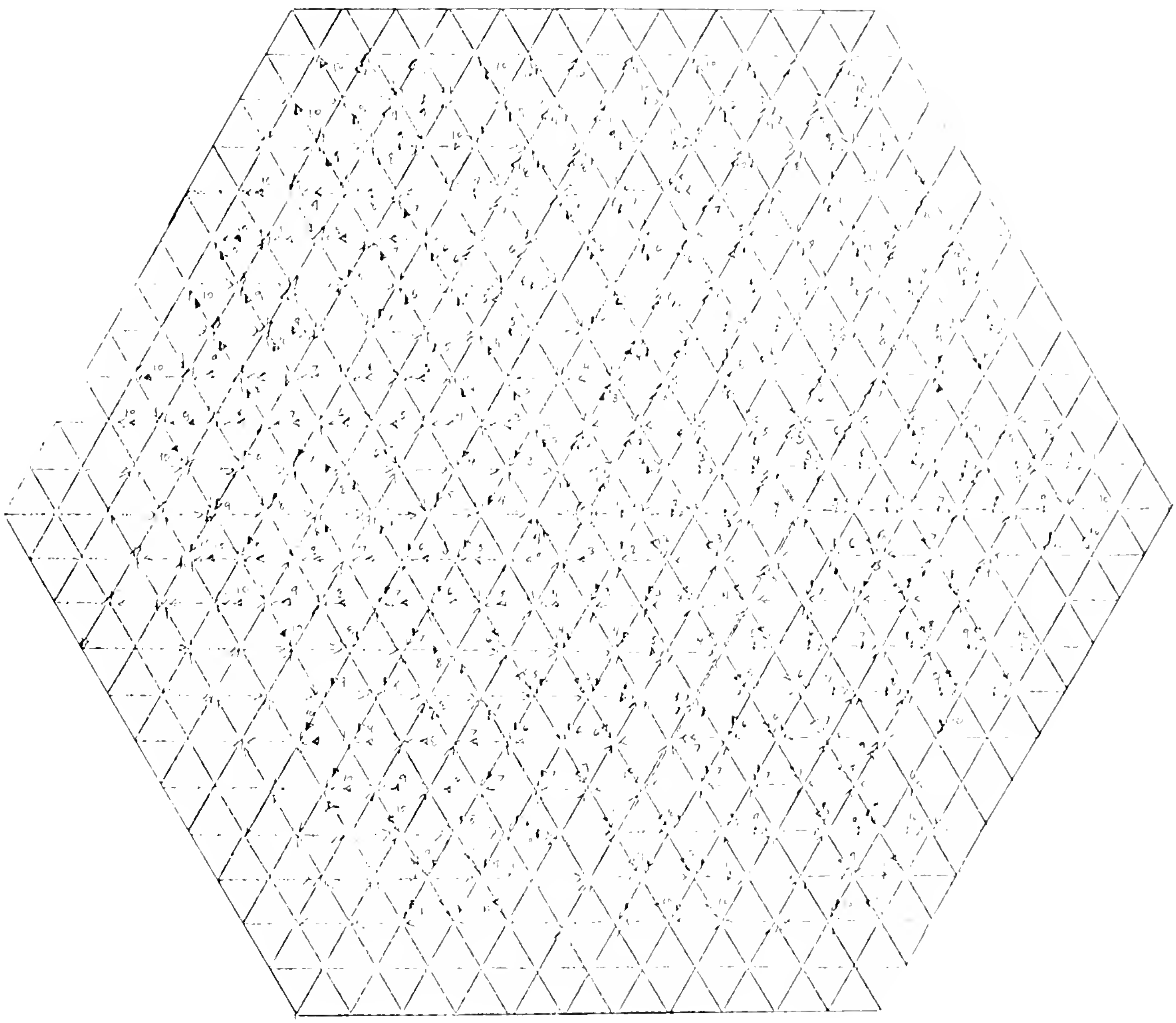


Figure 4.7 General Message Propagation in Uni-directional Hexagonal Class K+D Type II Network.

Network	I		I+D		I+D+K		D+K Type I		D+K Type II		K		Mixed	
Time	Rate	Cum.	Rate	Cum.	Rate	Cum.	Rate	Cum.	Rate	Cum.	Rate	Cum.	Rate	Cum.
1	3	3	3	3	3	3	3	3	3	3	3	3	2	2
2	8	11	7	10	7	10	9	12	7	10	6	9	7	9
3	13	24	12	22	12	22	15	27	12	22	9	18	12	21
4	17	41	17	35	17	39	27	54	18	40	12	30	18	39
5	22	63	21	60	22	61	21	75	23	63	15	45	23	62
6	26	89	24	84	30	91	45	120	34	97	18	63	28	90
7	31	120	30	114	36	127	27	147	38	135	21	84	34	124
8	35	155	34	148	38	165	63	210	42	177	24	108	40	164
9	40	195	36	184	52	217	33	243	46	223	27	136	46	210
10	44	239	43	228	55	272	81	324	60	283	30	166	49	259

Table 4.1 Propagation Statistics

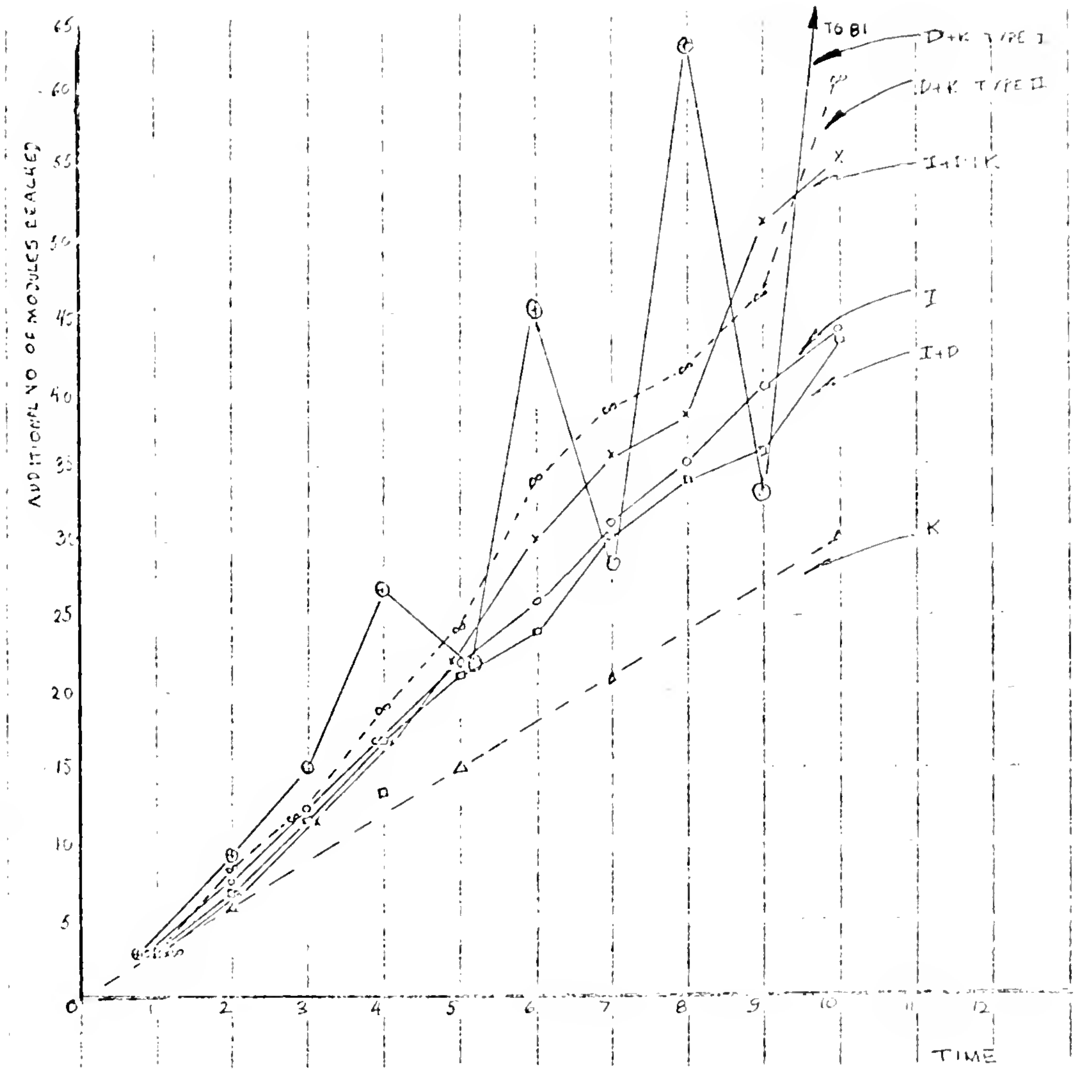


Figure 4.8 Propagation Rates in Some Uni-directional Hexagonal Networks.

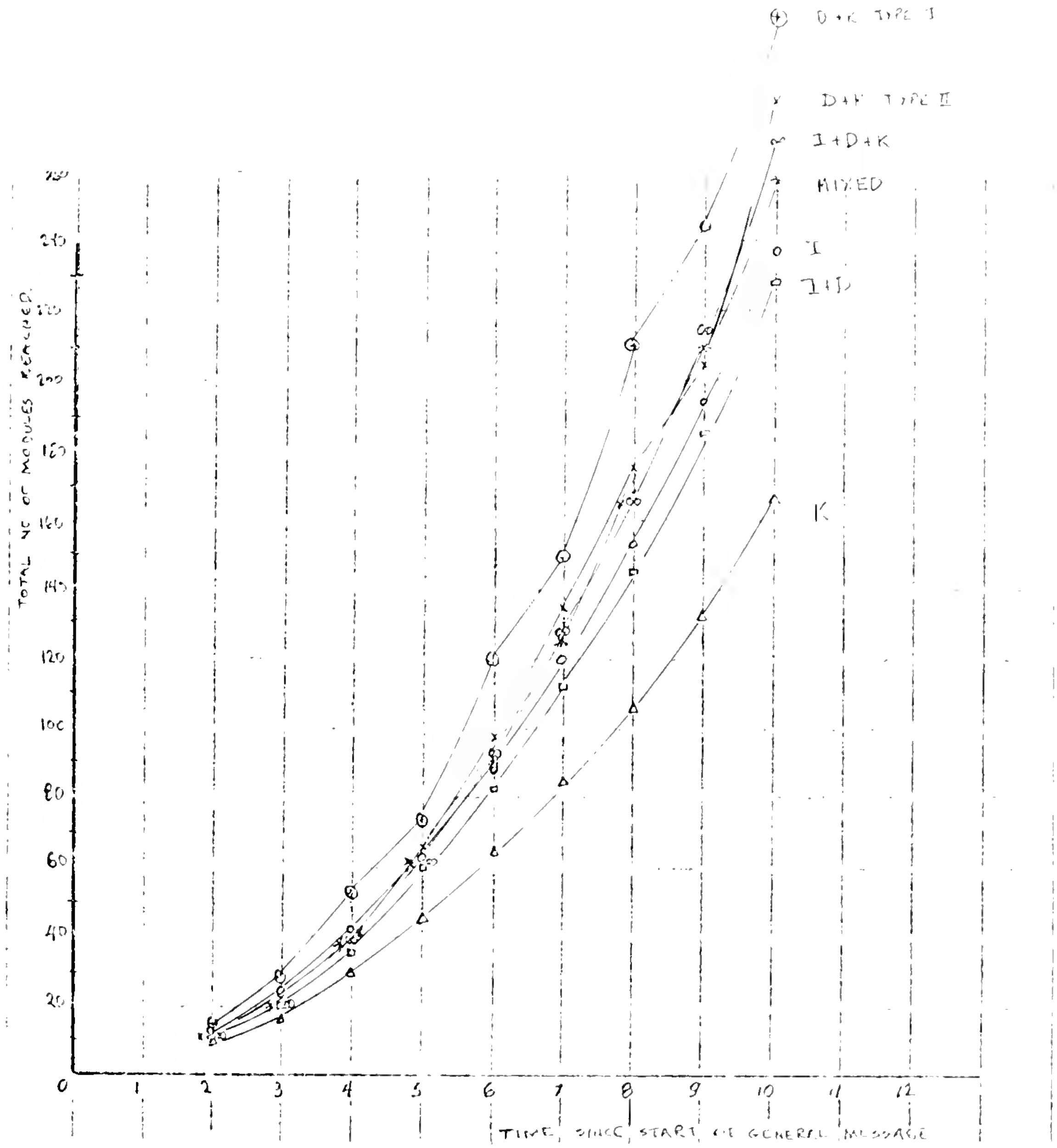


Figure 4.9 Cumulative Propagation in Some Uni-directional Hexagonal Networks.

of a given uni-directional network. Assuming that straight-flow and adjacence of channels of the same direction are the only important factors, we can define a network ranking function which is a function of indexes of flow and orientation. For orientation let us assign a point for each channel that is adjacent to a channel having the same direction. Hence a network of class K would have an orientation index of zero while class I network would have an index of four. The maximum value of this index is six.

Defining a flow index is more difficult. Assigning a point for each direction in which straight flow is allowed is not enough. We have to differentiate between a network which allows flow in all directions but with three lines directed the same way, the next three lines directed in the opposite direction; and the network which again allows flow in all directions but in which one line points one way and the other in the opposite way. The following index rule appears satisfactorily discriminatory and also simple: assign one point for a line that allows straight flow in one direction. Divide this by the minimum number of modules that must be traversed to encounter a line pointing in the same direction. Do this for all directions of flow allowed. Then add the points. Maximum value is six.

Notice that while the orientation index of a module can be determined by looking at the module alone, flow index is determined by the entire network. So one index is a measure of channel arrangement at the module level, while the other reflects the arrangement of the network.

When a network contains modules with different indexes, a composite index can be computed by weighing each modular index by the relative frequency of that module and summing the result.

The orientation and flow indexes computed according to such rules appear in table 4.2.

Network Type	Efficiency Order	Orientation Index	Flow Index
3D+K Type I	1st	$(3x6+0)/4 = 4\frac{1}{2}$	6
3D+K Type II	2nd	$(3x6+0)/4 = 4\frac{1}{2}$	$6(\frac{1+\frac{1}{2}}{2}) = 4\frac{1}{2}$
12I+3D+K	3rd	$(12x4+3x6+0)/16 = 4\frac{1}{8}$	$6(\frac{1}{4}) = 1\frac{1}{2}$
I	4th	4	0
3I+D	5th	$(3x4+0)/4 = 3$	$3(\frac{1}{2}) = 1\frac{1}{2}$
K	6th	0	3

Table 4.2 Orientation and Flow Indexes

Let f, the composite index, be: $f = ax^m + by^n$

where

x = orientation index

y = flow index

By substituting the values of the indexes we have:

$$f(D+K)\ \text{Type I} = a4.5^m + b6^n$$

$$f(D+K)\ \text{Type II} = a4.5^m + b4.5^n$$

$$f(I+D+K) = a4.125^m + b1.5^n$$

$$f(I) = a4^m$$

$$f(I+D) = a3^m + b1.5^n$$

$$f(K) = b3^n$$

Order of the equations indicate the efficiency ranking empirically determined (see the plots of cumulative propagation). We immediately see that:

$$a4^m > a3^m + b1.5^n$$

which implies that

(1) $a4^m - a3^n > b1.5^n$

and

$$a3^m + b1.5^n > b3$$

which implies that

(2) $a3^m > b3^n - b1.5^n$

From these conditions alone we cannot determine a, b, m, n. However, a linear relationship may be a good first approximation. So let $m = n = 1$.

Then from (1), and (2) respectively we get

$$a > 1.5b$$

$$a > .5b$$

The first condition, i.e. $a > 1.5b$ dominates. Since $a=2$, $b=1$ satisfy this condition for simplicity I have chosen those values. Hence we have

$$f = 2x + y$$

As was argued before, orientation is more important than flow, in fact at least one and a half times as important.

It goes without saying that $f = 2x + y$ is a very rough statement and that it does not take into account possible interdependence of x and y. It can only be considered a starting point to motivate future research to be based on the construction of multitude of networks.

CONCLUDING COMMENTS

Conclusions

This research aimed to determine the structure and the operating rules of networks that allow intermodular communication without resort to addressing. Addressing is defined here as the capacity to locate a module or a storage site irrespective of its contents.

While it was clear from the beginning that networks having two-way channels would achieve intermodular communication, this was not clear for the networks in which channels were one-way, being directed either towards or away from a module.

Investigations of square and hexagonal arrays of modules with one-way channels (uni-directional networks) reveal that for some arrangements of one-way channels the propagation speeds are about the same as the same array with two-way channels. Adjacence of one-way channels directed the same way improves propagation speed. For instance networks containing only type modules are twice as fast as networks containing only type.

In a network when a module wants information which might be contained by anyone of the modules, the information request or the general message must be propagated to the entire network. This is why propagation speed is important. The basic question of this research was: After the general message is propagated can the module or modules having the appropriate information send a response and have it routed so that it reaches the general message source a) without knowing where the source is, b) without the modules on the

response path relying for routing decisions on what the other modules have done and will do, and c) without cycles in the response path? It is clear that response routing of this kind is possible in bi-directional networks; the response simply follows a propagation path in reverse.

It is shown in this study that response routing is also possible in networks with one-way channels. The rules, to be applied by a module independently of and in ignorance of what other modules are doing, are very simple.

In the uni-directional networks the length of the response path between the source of the request and a responding unit changes with the arrangement of channels. It appears that the adjacence of channels with the same orientation shortens the response path. For instance in the square the type module which improves propagation speed also improves response time. In fact propagation and response times differ only by a constant from the bi-directional times. Hence the ratics of cumulative propagations and response times in the two types of networks approach 1.0 very rapidly as the size of the network grows.

The uni-directional networks accomplish general message propagation and response routing tasks with less memory, less logic, and fewer computations. Therefore, when the uni-directional network is such that response and propagation times are about the same as in the bi-directional one, having one-way channels would be preferable to having two-way channels.

The efficiency, defined as the ratio of cumulative propagations in the one-way and two-way channel arrangements, appears to be influenced by

a) the channel arrangement (as noted before) and b) the extent of straight flow provided. The channel arrangement while influencing straight flow does not determine it because of the possibility of mixing modules of different channel arrangements.

Neither factor alone is entirely responsible for efficiency. Both the extent of the adjacency of channels of the same direction, and the extent of straight flow provided directly correlate with propagation speed.

Implications

For understanding the workings of the brain, the results of this research are important because a neuron is a "module" with one-way channels - the dendrite carries messages towards the neuron and the axon away from it. Further more a significant number of neurons in the cortex exhibit "adjacence of channels having the sameflow direction." In this research adjacence was shown to contribute importantly to both propagation and response efficiency. It is also significant that only some of the uni-directional networks are efficient. Therefore, if it is postulated that the brain is an efficient organism this is equivalent to postulating that it is not a randomly connected net.

The results are pertinent to efforts towards building distributed-logic distributed-memory computers, i.e. modular computers. It appears that in such computers exhaustive search, that is the propagation of a message to the entire network can be accomplished reasonably rapidly. What is more important, after the exhaustive search, directed communication

can take place through very simple rules. Confining channels to one direction of transmission and arranging them suitably requires even less memory, less logic, and less computation.

When the problem at hand is intercomputer communication, once again, making channels one way and arranging them appropriately would solve the problem with less memory, less logic, and less computation than the bi-directional schemes; and in about the same time.

Further Research

Of necessity this research dealt with very simple networks and did not develop some of the issues. Further research in this area might address the following:

1. Susceptability of each type of network to malfunctions might be investigated. In particular, ability to adapt to local as well as to massive failures should be studied.

2. The behavior of networks constructed by mixing modules of different classes could be examined more thoroughly. The case of the hexagonal K+D network indicates that effort in this direction may be fruitful.

3. Research should be extended to three dimensional arrays. This would provide a much richer data base.

4. Arrays with connection patterns which do not necessarily confirm to a regular geometric pattern need to be studied.

5. The advantages of having both one-way and two-way channels in the same network must be considered.

6. Networks should be carefully examined when there is more than one module emanating a general message.

7. All the above studies should be done assuming that transmission takes a unit time and assuming that transmission time in part depends on the state of the module. In conjunction with this the case of delayed arrivals of the same general message should be looked at.

8. Determinants of uni-directional network efficiency should be more thoroughly researched.

9. After a cut in the network the conditions and rules necessary for a uni-directional network to reconnect itself could be determined.

10. Finally uni-directional networks should be given more complicated tasks like recognizing certain patterns or like forming an intercommunicating subnetwork;* and their abilities for accomplishing these tasks shuold be studied.

Modular arrays and especially those having one-way channels appear to be a very challenging and fruitful area of research.

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*A forthcoming working paper will discuss the author's research on recognizing rectangles, triangles, and circles using the hexagonal class K uni-directional network.

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